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PREFACE

Electrical networks constitute an essential element for ensuring a reliable and high-quality energy supply. Their study occupies a central place in the training of students in electrical engineering.

This document is designed as a course material intended mainly for third-year bachelor students in Electrical Engineering. Its objective is to present, in a progressive and structured manner, the fundamental concepts related to electrical networks, combining both theoretical and practical aspects. Emphasis is placed on understanding concepts, mathematical derivations, and the application of analysis tools. Although it is not exhaustive, this material provides a solid foundation for learning and deepening knowledge of electrical systems.

The document is organized into five chapters:

- **Chapter 1:** General overview of the power system and introduction to the basic concepts related to the structure of the electrical network.
- **Chapter 2:** Modes of transmission, allocation, and distribution of electrical energy, including the presentation of different topologies and network architectures.
- **Chapter 3:** Study of transmission line parameters, including longitudinal characteristics (resistance, inductance) and transverse characteristics (capacitance, conductance), as well as the calculation of power flow and voltage drops.
- **Chapter 4:** Transformers: their modeling, determination of parameters, coupling modes, and main types, with emphasis on the per-unit system.
- **Chapter 5:** Analysis of balanced and unbalanced faults, and presentation of the symmetrical components technique, which simplifies the study of unbalanced circuits.

Chapter I

Generalities on Electrical Networks

I.1. Introduction

Electrical energy plays a fundamental role in the development of modern societies, whether in improving living conditions or supporting industrial activities. It is therefore essential to produce electricity efficiently and continuously. To meet the growing demand, power plants have been established to generate large amounts of energy. Once produced, this energy must then be transmitted and distributed to consumers.

I.2. Historical Background on Energy Production and the Evolution of Electrical Networks

- **Before electricity:** energy came from wood, coal, water, and wind. It was used locally (mills, steam engines) with power outputs of only a few kilowatts (kW).
- **1879 – 1882 : The first power plants**

Thomas Edison's first power plant in New York (1882) operated with direct current, producing about 33 kW, mainly intended for public and private lighting.

- **The "War of Currents" (1880 – 1895)**

L'introduction The introduction of alternating current by Tesla and Westinghouse made it possible to increase power levels and, above all, transmission distances. The first AC lines already reached several megawatts (MW).

- **Early 20th century: Expansion of networks**

Coal-fired power plants multiplied. Capacity grew from a few MW to plants capable of supplying 50 to 100 MW each. Networks began to interconnect.

- **1920s – 1950s: Development of hydropower and high-voltage lines**

Large dams appeared, such as Hoover Dam (United States, 1936) with a capacity of 2,000 MW. High-voltage transmission (up to 220 kV and later 400 kV) made it possible to deliver hundreds of MW over long distances.

- **1950s – 1970s: The rise of nuclear power**

From the 1970s onward, nuclear power plants produced unit capacities ranging from 500 to 1,000 MW per reactor, with some sites exceeding a total of 3,000 MW. National grids became interconnected to handle these large power levels.

- **1980s – 2000s: Diversification and scaling up**

Coal- and gas-fired power plants reached several gigawatts (GW). Hydropower continued to expand (Aswan Dam: 2,100 MW; Itaipu Dam: 14,000 MW). At that time, the first wind turbines did not exceed 100 kW, and solar power plants only a few MW.

- **Since 2000: Energy transition and smart grids**

Renewables have experienced rapid growth. Onshore wind turbines now reach 5 to 7 MW, and offshore units exceed 12 MW. Large solar power plants surpass 1,000 MW (Bhadla Solar Park in India: 2,245 MW). Today, electrical networks must manage production ranging from small household generators (a few kW) to vast sites of several gigawatts.

I.3. Organization of the Electrical Network

The electrical network encompasses all the infrastructures that ensure the transmission of energy from the production centers (hydropower, thermal, nuclear, renewable, etc.) to the consumers (cities, factories, households). It is composed of lines operating at different voltage levels and interconnected through substations. These substations provide both the distribution of electricity and the transfer from one voltage level to another by means of transformers: high voltage for transmission to reduce losses, and then stepped down for final use.

An electrical network is organized into several hierarchical levels in order to deliver energy from production sites to end users:

1. **Generation**

   Electricity is generated in various power plants (thermal, hydro, nuclear, solar, wind, etc.) at high power levels.

2. **Transmission**

   The generated energy is transmitted over long distances through high-voltage (HV) and extra-high-voltage (EHV) lines, which helps reduce losses due to the Joule effect.

3. **Sub-Transmission (or Power Distribution at Intermediate Level)**
   Electricity is directed to large consumption areas through intermediate-voltage networks, via substations that step down the voltage.
4. **Distribution**
   Energy is distributed to users (industrial, tertiary, and residential) at medium voltage (MV) and then at low voltage (LV).
5. **Consumption**
   The end user (households, factories, services) receives electricity at the voltage level adapted to their needs (220/230 V for residential use, several kV for industrial use).

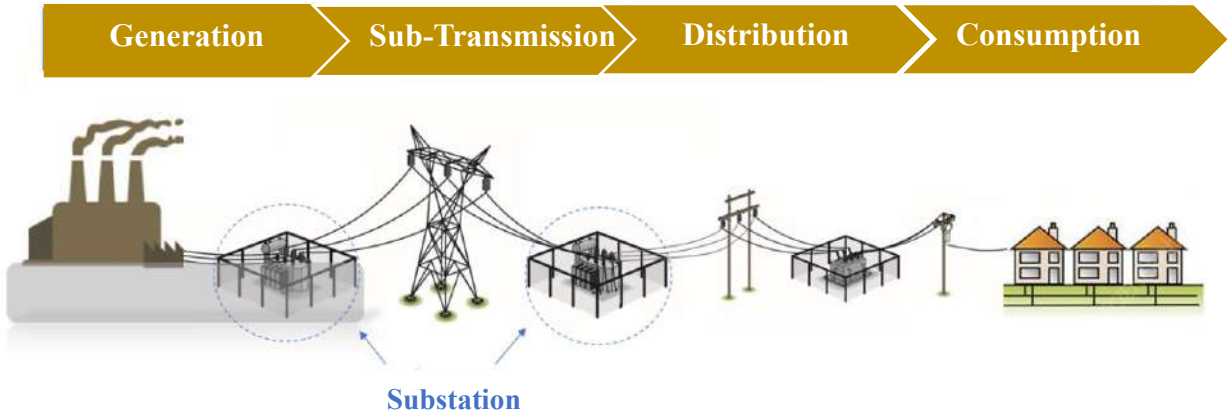


Figure I.1. Organization of an Electrical Network.

I.4. Power Plants

A power plant is an industrial facility designed to generate electricity from various primary energy sources (coal, gas, water, wind, sun, uranium, etc.). It converts mechanical, thermal, chemical, or light energy into electrical energy, most often by means of a turbine coupled to an alternator, except in the case of photovoltaic plants.

There are three main types of power plants, classified according to the energy source used:

- **Thermal power plants**: They produce electricity from fossil fuels (coal, gas, oil) or nuclear energy, by converting heat into mechanical energy, and then into electricity.
- **Hydroelectric power plants**: They harness the power of water (dams, tidal power plants) to drive turbines and generate electricity.**Renewable power plants**: They use natural energy sources directly, such as wind (wind turbines) or sunlight (photovoltaic and solar thermal).

I.4.1. Thermal Power Plants

Thermal power plants generate electricity from the heat released by the combustion of fossil fuels (coal, natural gas, oil) or from nuclear fission in the case of nuclear power plants. This heat is used to warm water and produce pressurized steam, similar to the principle of a pressure cooker. The force of this steam drives a turbine connected to a generator to produce electricity. After passing through the turbine, the steam is cooled and condensed back into water, which is then returned to the boiler.

To heat the water, several burners are installed in a boiler. These burners can be fueled by different types of fuel, such as oil, pulverized coal, or natural gas. The boiler consists of thousands of tubes through which water circulates. Under the effect of combustion, the water is heated to a high temperature and then sent to a drum where it is transformed into steam. This high-pressure steam is directed toward a turbine. As it moves, the turbine converts thermal energy into mechanical energy. It is coupled to a generator, which transforms this mechanical energy into electrical energy through the rotation of the rotor (the moving part) inside the stator (the fixed part). The electricity produced is then transmitted to the grid via a transformer.

They can be mainly classified into:

- **Conventional thermal power plants** (coal, gas, or oil);
- **Nuclear power plants**, which use the energy released by uranium fission;
- **Combined-cycle thermal power plants**, which improve efficiency by combining a gas turbine with a steam turbine.

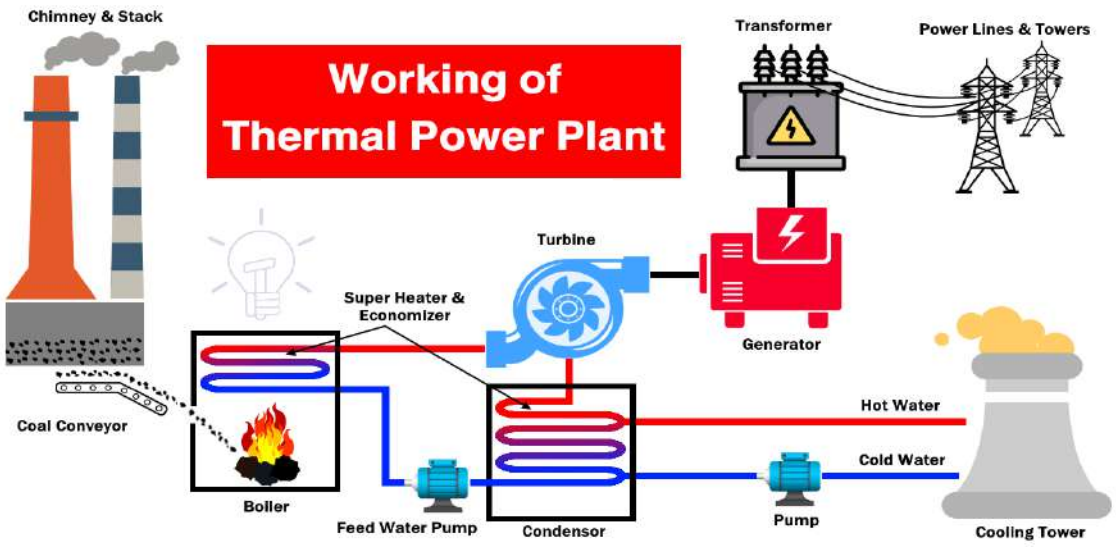


Figure I.2. Thermal power plant.

I.4.2. Nuclear Power Plants

Nuclear power plants operate on the same principle as steam thermal power plants, but the boiler is replaced by a nuclear reactor. In this reactor, heat is produced through nuclear fission, which is the splitting of the nuclei of uranium or plutonium atoms. The released energy heats a fluid that generates steam, which drives a turbine coupled to a generator, thus producing electricity. The steam is then condensed and returned to the circuit to repeat the cycle. The mechanical and electrical components (turbine, generator, condenser) are therefore similar to those of a conventional thermal power plant. The overall efficiency is generally between 30% and 40%.

However, these plants require a significant cooling system, which often necessitates locating them near a water source or the presence of a cooling tower. Finally, they produce radioactive waste and carry a potential risk of accidents.

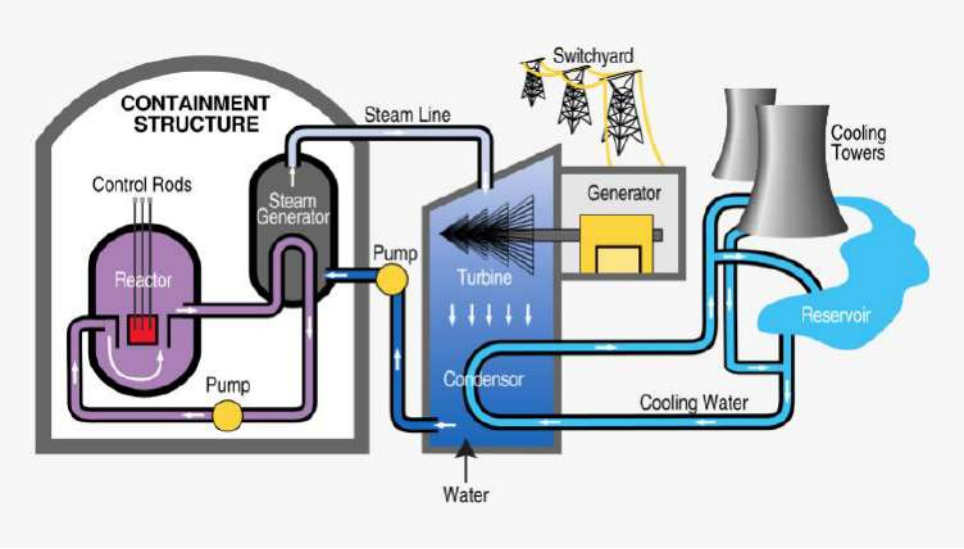


Figure I.3. Nuclear Power Plants.

I.4.3. Wind Power Plants

A wind turbine, or aerogenerator, is a device that captures the kinetic energy of the wind and converts it first into mechanical energy through its blades connected to a drive shaft, and then into electrical energy via a generator. The energy efficiency and power output mainly depend on wind speed. Within the operating range of a wind turbine, the generated power is approximately proportional to the cube of the wind speed, which highlights the importance of

site selection. A control system ensures a constant rotation speed compatible with the grid frequency (50 Hz) (Fig. 4).

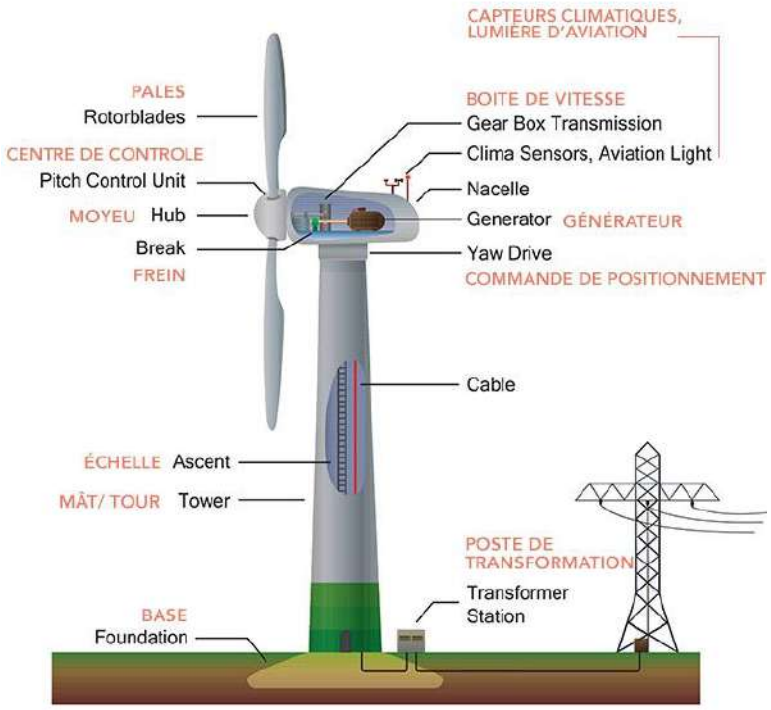


Figure I.4. Wind Power Plants.

I.4.4. Solar or Photovoltaic Power Plants

Photovoltaic solar energy is produced from solar radiation using photovoltaic cells, which are the basic components. These cells use the photoelectric effect to directly convert light into electricity. Since each cell provides a low voltage, they are assembled to form solar modules, which are then grouped to create photovoltaic panels. The electricity generated can be consumed on-site, stored in batteries, or fed into the electrical grid.

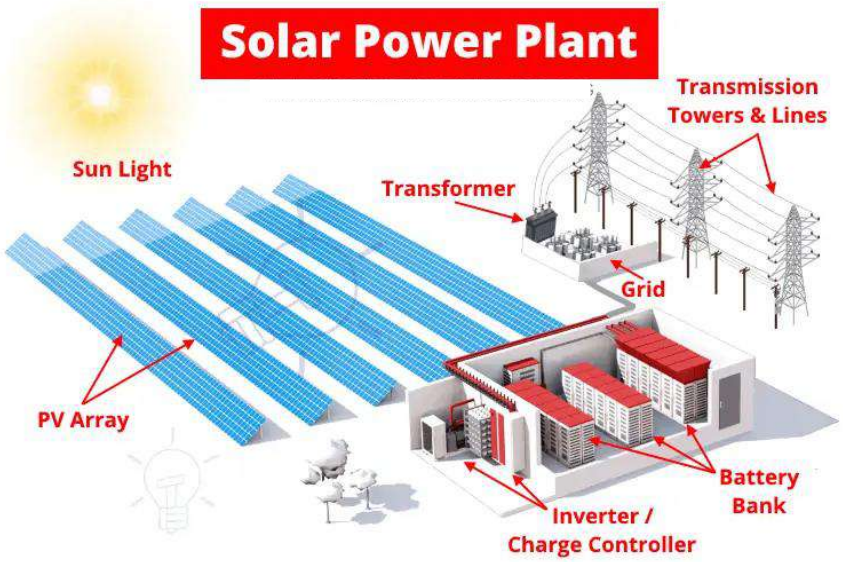


Figure I.5. Photovoltaic Power Plants.

I.4.5. Hydroelectric Power Plants

Hydroelectric power plants harness the potential energy of moving water to generate electricity. Water, released from a reservoir or river, drives a hydraulic turbine that converts this energy into mechanical energy. The turbine is coupled to a generator, which transforms the mechanical energy into electrical energy (Fig. 6).

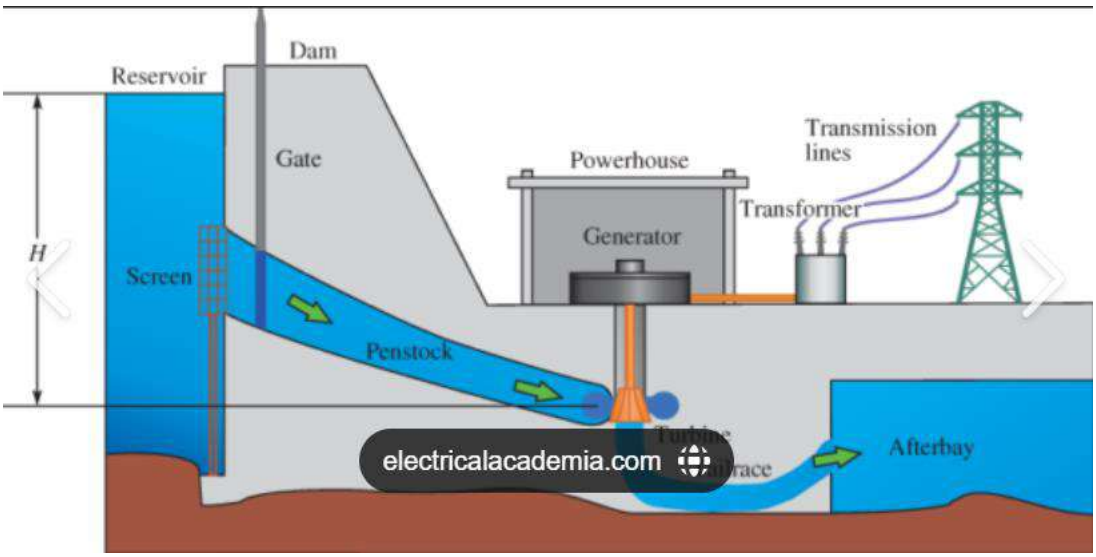


Figure I.6. Hydroelectric power plant.

I.5. Electrical Substation

Electrical substations are essential components of the power grid. They play a role in both the transmission and distribution of electricity. Their main function is to step up the voltage for

long-distance transport and then step it down to match the consumption needs of end users, whether residential or industrial. Located at the ends of transmission or distribution lines, these substations—often simply called “substations”—perform several functions: adjusting voltage levels to the characteristics of lines and networks, directing electricity toward different lines, and providing remote monitoring of the electrical flow.

There are mainly three types of electrical substations based on their functions:

- **Interconnection substations**, which provide the link between different parts of the transmission network.
- **Transformation substations**, which adjust voltage levels between transmission and distribution.
- **Hybrid substations**, which combine the two previous functions.

I.6. Main Components of an Electrical Substation

An electrical substation consists of several pieces of equipment that allow for the transformation, protection, control, and routing of electrical energy (Fig. 7).

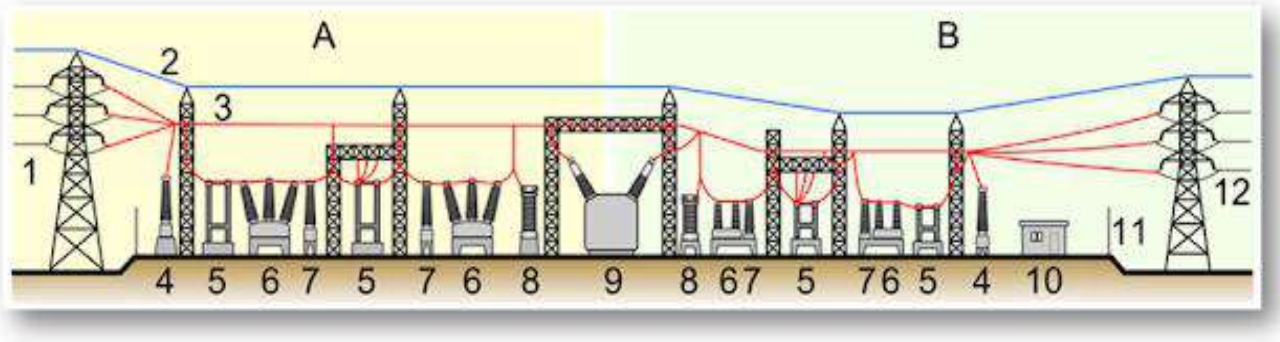


Figure I.7. Electrical equipment in a substation (A: primary side, B: secondary.

The main components are:

1. **Primary power line**: high or extra-high voltage line that brings electricity to the substation.
2. **Shield wire**: cable located above the lines to protect against lightning.
3. **Power line**: main conductor that transports electrical energy.
4. **Voltage transformer (VT)**: reduces voltage to a measurable value for protection and metering systems.

5. **Disconnector (isolator)**: visible switching device that allows part of the circuit to be electrically isolated (not operable under load).
6. **Circuit breaker**: protective device that automatically interrupts the current in case of overload or short circuit.
7. **Current transformer (CT)**: reduces current intensity for measurement and protection purposes.
8. **Surge arrester**: protects the installation against overvoltages caused by lightning or switching operations.
9. **Power transformer**: steps voltage up or down to adapt it for transmission or distribution.
10. **Control building**: houses control, protection, and monitoring equipment (relays, SCADA, batteries, etc.).
11. **Fence**: safety element preventing unauthorized access to the substation.
12. **Secondary power line**: outgoing line that redistributes electricity to the transmission or distribution network.

I.6.1. Power Transformer

The power transformer is undoubtedly the equipment that made the rise and widespread adoption of alternating current (AC) electrical networks possible. It has enabled the large-scale transmission, distribution, and utilization of electrical energy.

Its main function is to modify the electrical voltage:

- **Step-up voltage**: at the output of power plants, voltage is increased (for example, from 20 kV to 400 kV) to reduce current and, consequently, Joule losses in transmission lines.
- **Step-down voltage**: close to consumers, voltage is reduced (for example, from 63 kV to 20 kV) to safely supply distribution networks.



Figure I.8. Power Transformer.

I.6.2. Measurement transformers (Current and Voltage)

Les transformateurs de mesure sont utilisés lorsqu'il est nécessaire de mesurer des tensions ou des courants trop élevés pour être directement appliqués aux appareils de mesure. Leur rôle principal est de réduire proportionnellement la grandeur électrique à mesurer (tension ou courant) tout en assurant une isolation galvanique entre le réseau haute tension et les instruments de mesure ou de protection. Ils doivent effectuer cette transformation sans déphasage notable. On distingue deux types principaux :

- ✓ **Voltage Transformers (VT):**

Voltage transformers are primarily used on high-voltage lines to supply measuring devices (such as voltmeters and wattmeters) or protection equipment (relays). Their role is to reduce the network voltage to a standardized value, usually around a few hundred volts, and to protect instruments from high voltage by providing electrical isolation.

- ✓ **Current transformer (CT) :**

Current transformers are used to reduce the high currents flowing in high- or low-voltage lines to a value that can be easily measured. The primary winding of the CT is connected in series with the line whose current is to be measured, enabling accurate and reliable measurement. These transformers are designed exclusively for measurement and protection purposes, and their power is generally low, ranging from 15 to 200 VA. The rated current of their secondary winding is standardized between 1 A and 5 A, which facilitates connection to measurement, metering, and protection devices.



(a) Current transformer (CT) (b) Voltage Transformers (VT)

Figure I.9. Measurement transformers

I.6.3. Circuit breaker

The circuit breaker is an essential protective device in an electrical network. Its main role is to protect installations against overloads and fault currents, such as those caused by lightning or accidental contact with obstacles (e.g., a tree branch). According to the definition of the International Electrotechnical Commission, a circuit breaker is designed to establish, withstand, and interrupt currents at its rated voltage, i.e., the maximum voltage of the network it protects. It operates both under normal operating conditions, such as connecting or disconnecting a line, and under abnormal conditions, such as when clearing a short circuit. Thanks to its characteristics, it is an essential protective device for high-voltage networks, as it is the only device capable of effectively interrupting a short-circuit current and thus preventing serious damage to connected electrical equipment.



Figure I.10. circuit breaker.

I.6.4. Disconnect switches

A disconnector is an electromechanical device used in electrical substations to ensure the visible disconnection of a circuit and direct the current within the network. Unlike a circuit breaker, which isolates but does not physically separate, a disconnector guarantees effective mechanical separation, thus providing sufficient separation distance to ensure electrical safety. Its main function is to take an element of the network, such as a high-voltage line, a transformer, or part of a substation, out of service so that an operator can safely perform maintenance operations. It can also be used to isolate a faulty part of the network and keep the rest of the installation in operation (Figure I.11).



Figure I.11. Disconnect switches.

I.6.5. Busbars

In electrical distribution, a busbar is a copper or aluminum conductor that conducts electricity within an electrical panel, switchgear, or substation. It can take the form of flat bars or hollow tubes, which are considered low-impedance conductors that can connect multiple circuits at different points. In high-voltage applications, busbars are generally installed on insulators when they are rigid, or stretched flexibly and suspended from metal structures by insulating chains. They play an essential role in connecting the various components of an electrical substation, but their fragility is a major weakness: in the event of a short circuit on a busbar, the entire substation is taken out of service. To limit this risk and facilitate maintenance, several busbars are often connected in parallel, ensuring continuity of service even when work is being carried out on one of them (Fig. I.12).

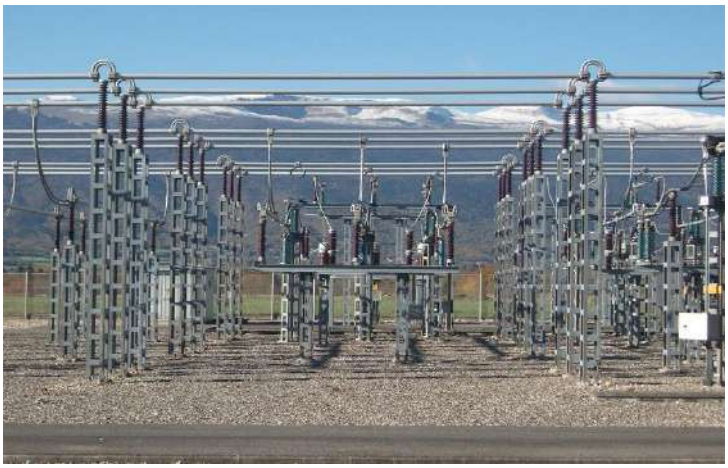


Figure I.12. busbar.

I.4.7. Insulators

Insulators are electrical components used to secure, maintain, and support bare electrical conductors while ensuring electrical insulation between them and their supports. They are mainly found on high-voltage lines and in electrical substations. In addition to their insulating role, they perform an essential mechanical function by supporting the weight of the conductors and climatic stresses (wind, snow, frost) (Figure I.13). On the transmission network, insulators are arranged in chains, the length of which depends on the voltage level:

Voltage (KV)	63 KV	90 KV	225 KV	400KV
Number of insulators	6	9	12	19

Their two main functions are:

- Prevent the electrical current flowing in the phase conductors from passing through the towers.
- Ensure the mechanical attachment of the phase conductors to the supports (towers or substation structures).



Figure I.13. Insulators.

I.4.7. Other substation equipment

- **Supports**

The role of electrical pylons is to maintain the conductor cables at a minimum safe distance from the ground and surrounding obstacles, in order to ensure the safety of people and the protection of nearby installations. The choice of pylon type depends on the characteristics of the line to be built, the environment it crosses, and the mechanical constraints imposed by the terrain and local climatic conditions. Their shape and silhouette are determined by the layout of the conductor cables. In general, towers are constructed as steel lattice structures. The higher the voltage of the line, the greater the span of the towers and the greater their height must be.

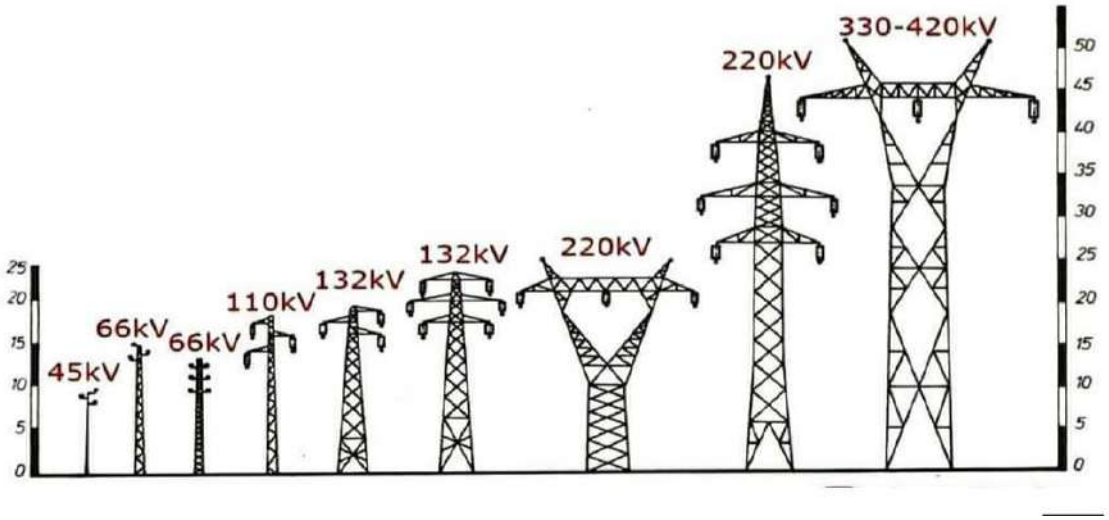


Fig I.14. Some pylon structures.

- **Conductor cables**

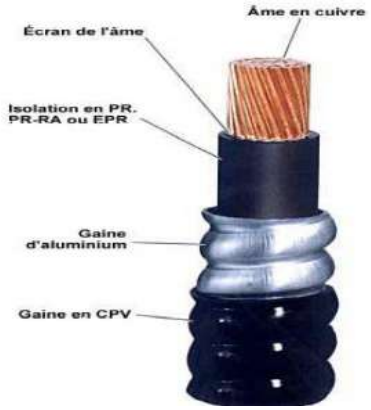
There are two types of electrical cables used for energy transmission and distribution: overhead and underground.

- Overhead cables, which are generally bare, are the visible transmission lines. They are electrically insulated by rigid or suspended insulators attached to poles or pylons anchored to the ground by foundations. These cables are composed of layers of aluminum conductor strands, often reinforced with a steel core to increase their mechanical strength, or made of aluminum alloy (Almélec) (Figure I.15).
- Underground cables, which are more expensive and mainly used in urban areas, consist of aluminum or copper conductors, twisted into a helix and covered with insulation. This insulation can be oil-impregnated paper or, increasingly, synthetic materials. Finally, they

are protected externally by metal sheaths (lead or aluminum), which are themselves covered with synthetic sheaths to provide greater mechanical and chemical resistance (Figure I.16).



Figure I.15. Overhead line conductor



FigureI.16. Cutting an underground cable

- **Guard cables**

Guard wires do not carry electrical current. Placed above high-voltage overhead line conductors, their main role is to provide protection against lightning by intercepting discharges and preventing the main conductors from being struck directly. Most often made of steel, they have high mechanical strength, enabling them to withstand the stresses caused by weather conditions and mechanical forces (Figure I.17).



Figure I.17. Guard cables.

- **Lightning rod**

A lightning rod is a device designed to protect buildings and structures from direct lightning strikes by safely channeling electrical charges to the ground. Its role is not to attract lightning, but to capture it when it strikes nearby, preventing it from striking randomly and causing

significant damage. To ensure optimal protection, lightning rods are always installed on the highest part of the building or structure to be protected



Figure I.18. lightning rod.

- **Lightning arrester (or surge protector)**

A surge arrester is a protective device that limits power surges caused by lightning or switching operations. It works by diverting excess current to ground as soon as a power surge occurs. A surge arrester generally consists of an upper part connected to the conductor of the line to be protected and a lower part connected to a low-resistance earth connection (often less than 1 Ω) (Figure I.19).



Figure I.19. Surge protector.

- **Dispatch center**

The network control center, also known as dispatching, is a modern control room responsible for remotely managing the high- and medium-voltage power grid. It monitors the status of the grid in real time using computer systems (SCADA: Supervisory Control And Data Acquisition) that enable supervision, control, and data acquisition. These systems display voltage and power values as well as the position of equipment. Its main mission is to ensure a balance between electricity production and consumption, maintain the voltage plan, and ensure the safety of transit on the interconnected network. Operating 24/7, the dispatching center enables rapid intervention in the event of a failure and guarantees the continuity of the power supply, particularly for distribution networks and high-voltage consumers.

Fonctionnement du SCADA

- **Supervision:** real-time monitoring of substations, lines, and equipment.
- **Control:** ability to operate remotely (open/close a circuit breaker, adjust voltage, etc.).
- **Data acquisition:** automatic collection of information on voltages, power, currents, and device statuses.
- **Alarms:** instant notification of network anomalies or faults.
- **History:** recording and analysis of data to optimize operation.



Figure I.20. Dispatch center.

Chapter II

Modes of transport, transmission, and distribution of electrical energy

II.1. Introduction

The transmission and distribution of electricity play a crucial role in the operation of power systems. Indeed, the electricity generated in power plants must be transmitted over long distances to consumers (industrial, commercial, and residential) with minimal losses and in compliance with quality standards. To maintain the balance between electricity generation and consumption, thereby ensuring economic performance, the electrical infrastructure of a country is generally organized into several levels, each corresponding to a specific type of power network.

II.2. Description of Power Networks

An electrical network is a set of infrastructures (lines, cables, transformers, substations, and control equipment) that enable the transmission of electricity from generation plants to end-users. It is organized into several voltage levels — transmission (High Voltage, HV and Extra High Voltage, EHV), sub-transmission (High Voltage, HV and Medium Voltage, MV), and distribution (Medium Voltage, MV and Low Voltage, LV) — in order to ensure a reliable, continuous, and demand-oriented supply.

II.3. Structure of Electrical Networks

An electrical network consists of four stages: generation, transmission, sub-transmission, and distribution of electricity to end-users.

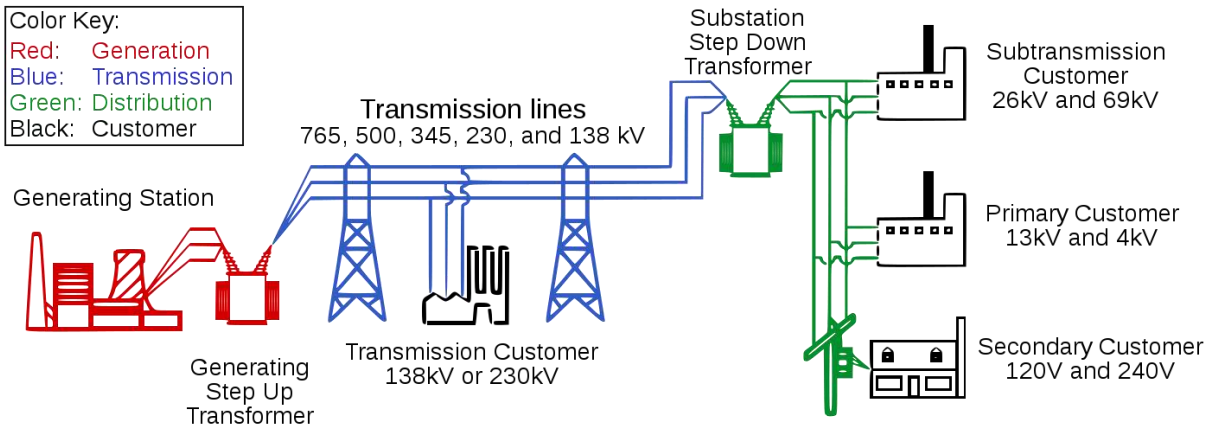


Figure II.1. General Structure of Electrical Networks.

II.3.1. Power generation

Electricity generation consists in producing the active power required to meet the total demand of the network. In most cases, this electricity is generated in three-phase systems using alternators driven by various primary energy sources (natural gas, coal, water, wind, solar energy, etc.).En Algérie, la production repose principalement sur les centrales thermiques à gaz, complétées par des centrales hydrauliques et, plus récemment, par des unités renouvelables :

- ✓ Hadjret Ennous Thermal Power Plant (Tipaza): 1,200 MW (natural gas).
- ✓ Terga Power Plant (Aïn Témouchent): 1,200 MW (natural gas).
- ✓ Koudiet Eddraouch Power Plant (El Tarf): 1,200 MW (natural gas).
- ✓ Koudiat Acerdoune Hydropower Plant (Bouira): about 100 MW.
- ✓ Hassi R'mel Solar Power Plant (Ghardaïa): 150 MW (solar-gas hybrid).
- ✓ Adrar Wind Farm: 10 MW.
- ✓ Naâma Combined Cycle Power Plant (Touifza, Mecheria): with a capacity of 1,163 MW, it uses gas-steam technologies to optimize efficiency.
- ✓ Sedret-Leghzal Photovoltaic Solar Power Plant (Naâma): with a capacity of 20 MW, it contributes to energy diversification through renewables.

II.3.2. Transmission

The transmission of electrical energy consists in conveying the power generated by production units to consumption areas. Electricity is first generated by alternators at a medium voltage (12 to 15 kV), then injected into the transmission network through transformer substations that step it up to high or extra-high voltage levels, in order to minimize losses during transmission. The choice of voltage depends on both the distances and the power to be transmitted: the greater the distance and the power, the higher the required voltage. For instance, in Algeria the transmission network generally operates at 220 kV, with some lines reaching 400 kV in the southern regions. In Europe, transmission voltages reach 400 kV, while in North America they can be as high as 735 kV.

II.3.3. Sub-Transmission

Sub-transmission networks ensure the connection between the transmission network and the distribution networks. Their main role is to transfer electricity from transmission lines to major consumption centers through interconnection substations (Extra High Voltage, EHV / High

Voltage, HV or High Voltage, HV / Medium Voltage, MV). They directly supply large industrial consumers at High Voltage (HV) or Medium Voltage (MV) and distribute energy across urban and rural areas. These networks generally operate at High Voltage (HV) levels ranging from 30 kV to 150 kV (and in some countries up to 275 kV). In Algeria, sub-transmission is typically operated at 60 kV, with networks that are predominantly overhead, although underground lines are also used in large urban areas.

II.3.4. Distribution

Distribution networks are responsible for supplying all residential, commercial, and industrial consumers, delivering energy from the sub-transmission networks to the end-users.On distingue two sub-levels of voltage in the distribution:

- Medium Voltage (MV): generally ranging from 1 kV to 30 kV (commonly 10, 20, or 30 kV depending on the country). It is mainly intended for supplying medium-sized industrial customers, hospitals, shopping centers, and public distribution substations.
- Low Voltage (LV): below 1 kV (230/400 V in Algeria and Europe), used for the direct supply of residential consumers, commercial facilities, and small workshops.

The structure of distribution networks can be overhead (in rural and semi-urban areas, to reduce costs) or underground (in city centers, for safety reasons and urban integration)

II.4. Voltage Levels

The new standards in force in France (UTE C 18-510) and (NF C 15-100) define the levels of alternating voltage as follows:

Table II.1. Voltage Ranges

Voltage Levels		Voltage Value in Volts		Voltage Levels of the Algerian Power Grid
		In AC (Alternating Current)	**In DC (Direct Current)**	
Extra Low Voltage	**(ELV)**	$U \leq 50$	$U \leq 120$	--------
Low Voltage (LV)	**LVA**	$50 < U \leq 500$	$120 < U \leq 750$	380
	LVB	$500 < U \leq 1000$	$750 < U \leq 1500$	--------
High Voltage (HV)	**HVA(MV)**	$1000 < U \leq 50\,000$	$1500 < U \leq 75\,000$	10000/30000
	HVB	$U > 50\,000$	$U > 75\,000$	600000-220000-400000

II.5. Topology of Electrical Networks

The topology of an electrical network refers to the way in which lines, substations, and equipment are interconnected. It varies according to the type of network (transmission, sub-transmission, distribution) and depends on several technical and economic criteria. The choice of topology is mainly guided by :

- The desired level of reliability: a highly meshed network ensures better service continuity and limits outages in the event of a fault, unlike a radial topology which is simpler but more vulnerable.
- Flexibility and maintenance: certain structures allow easier isolation of a part of the network to carry out maintenance work, while still maintaining the supply to users.
- Investment and operating costs: a meshed or ring network offers greater security but requires more lines and substations, and therefore involves higher costs..
- Scalability: the ability of the network to integrate new loads, new generation sources (particularly renewables), or to adapt to urban and industrial growth.

II.5.1. Loop Network

The loop topology is mainly used in sub-transmission and medium voltage (MV) distribution networks. High Voltage/Medium Voltage (HV/MV) substations are interconnected to form loops, which allows supply to be maintained even in case of a fault or maintenance operation. However, not all MV networks are meshed: in rural areas, radial networks are often used (simpler but less reliable), while in urban or industrial areas, the loop configuration is preferred to ensure service continuity (Figure II.2.a).

II.5.2. Meshed Network

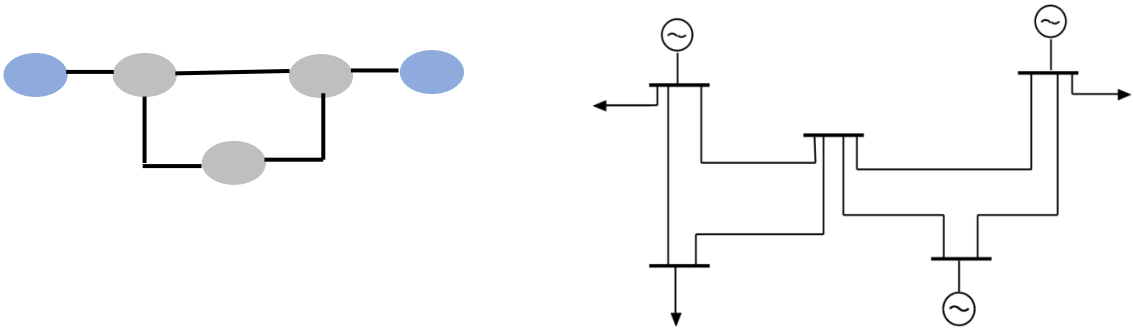
The meshed topology is the most widely used for transmission networks. All generation centers and Extra High Voltage (EHV) interconnection substations are interconnected, creating a highly integrated network. This structure ensures high reliability and secures the power supply in case of a line failure. However, it requires large-scale management and monitoring, often at the national or even continental level (Figure II.2.b).

II.5.3. Radial Network

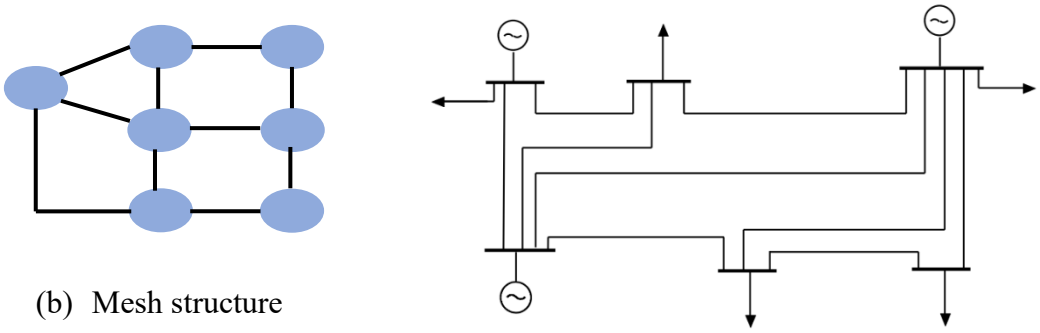
The radial topology is the most common in medium voltage (MV) and low voltage (LV) distribution networks. It consists of a main feeder supplied by a High Voltage/Medium Voltage (HV/MV) primary substation, which serves several distribution substations. In medium voltage (MV), this structure can be supplied from both ends in order to improve service continuity (Figure II.2.c).

II.5.4. Tree network

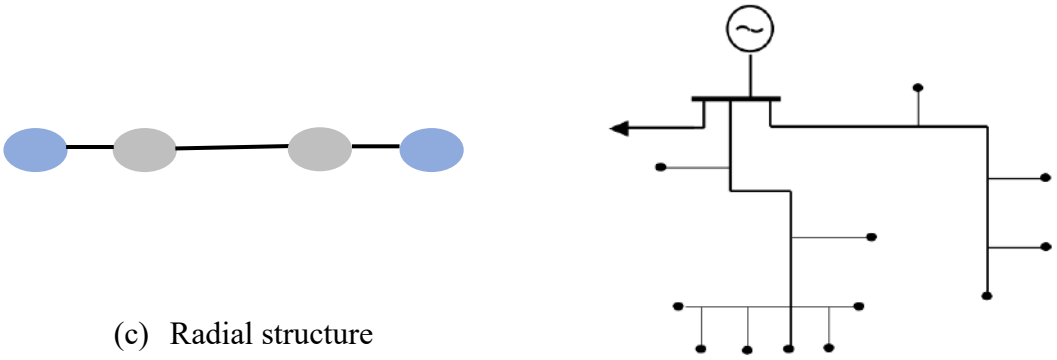
This structure is commonly used in rural areas, and sometimes in urban areas when loads are not highly sensitive to outages. It consists of a distribution substation that supplies several low-voltage (LV) substations through branches (taps) taken along the MV/LV lines. (figure II.2.d).



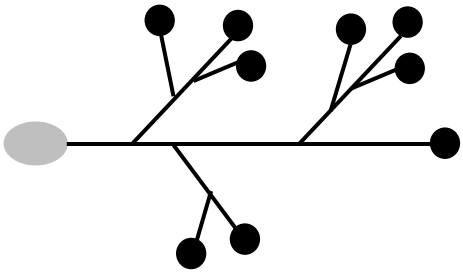
(a) Looped structure



(b) Mesh structure



(c) Radial structure



(d) Tree structure

Interconnection station MV distribution station Low-voltage distribution station

Figure II.2. Electrical network topologies.

II.6. Electrical substations

At the output of power plants, transformer stations raise the medium voltage generated to a high voltage level, which is essential for ensuring the economical transmission of electrical energy. This high voltage is then lowered again in other transformer stations located near consumption centers.

These substations thus constitute the junction points between the different voltage levels of the grid. In general, the electrical grid is organized hierarchically, in the direction of energy flow (Figure II.1), and comprises the following elements:

- **HV/MV or EHV/MV primary substations**: they are supplied by the transmission or sub-transmission network and provide the transformation to the medium voltage (MV) level
- **Medium Voltage (MV) network:** consisting of medium-voltage feeders originating from the primary substations, implemented as either overhead lines or underground cables.
- **MV/LV substations:** public distribution substations that step down the voltage for low-voltage (LV) supply.
- **Low Voltage (LV) network:** the final link in the network, directly supplying the connections to end-users.

⭘ Workstation architectures

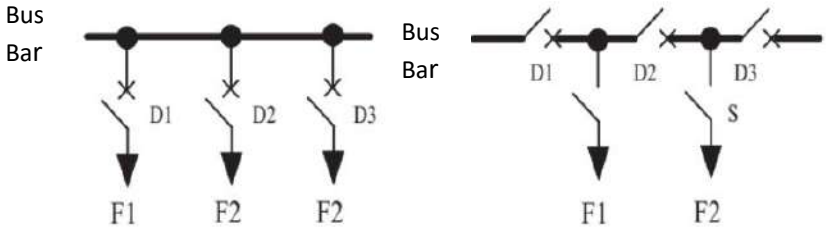
The choice of an electrical substation architecture is based on several techno-economic parameters, such as reliability, flexibility, ease of maintenance, as well as investment and operating costs.

The reliability and flexibility of a substation mainly depend on its internal architecture, which is defined by:

- The number and arrangement of busbars,
- The number and arrangement of switching devices (notably circuit breakers),
- And, if applicable, the feeding scheme of the substation (number and configuration of incoming lines).

From an architectural point of view, electrical substations can be classified into two main categories, regardless of their type:

- Bus-coupling substations: the busbars provide the coupling between the different feeders.
- Circuit-breaker-coupling substations: it is the circuit breakers that provide the coupling between the different feeders.



Bus Coupling Circuit-Breaker Coupling

Figure II.3. Substation Architectures

☞ **Example :**

From a reliability perspective, in a bus-coupling architecture, a fault on a feeder (e.g., F1) requires only the opening of its circuit breaker (D1), but during maintenance of this breaker, the feeder is out of service. In contrast, in a circuit-breaker-coupling architecture, isolating the same fault requires opening both D1 and D2, but the feeder F1 can remain supplied during maintenance thanks to the second breaker. This architecture is therefore more reliable, but also more costly, as it requires more circuit breakers for the same number of feeders.

II.6.1. High-Voltage Delivery Substations (HVDS)

These delivery substations generally handle powers exceeding 10 MVA. The installation of the delivery substation extends from:

- On one side, the connection point to the high-voltage distribution network (HV),
- On the other side, up to the low-voltage side terminal of the HV/MV transformer(s).

High-voltage delivery substations (HVDS) can be implemented according to several electrical configurations. In practice, the most commonly encountered schemes are:

1. **Single-Feeder Configuration**: a single HV supply directly feeds one or more HV/MV transformers. (figure II.4.a).

   ✓ **Principle:** the supply is radial, without backup or redundancy.
   ✓ **Characteristics:**

   - Simple to implement,
   - Low cost,
   - Easy to operate.

   ✓ **Limitation:** in case of a fault or interruption on the supply line, the entire substation is taken out of service, resulting in a complete outage of the loads served

2. **Double-Feeder Configuration:** two separate supplies provide backup in case one source is lost. (figure II.4.b).

   ✓ **Principle** :

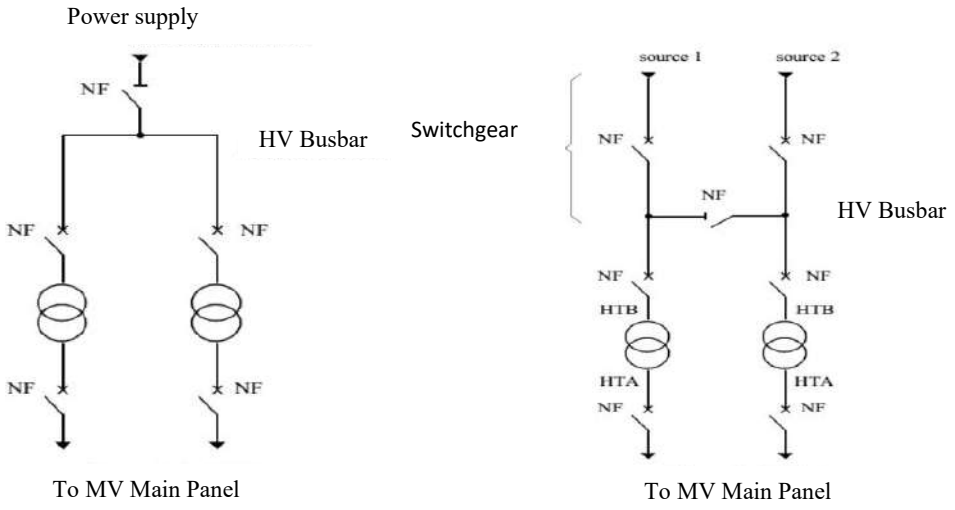
   Each HV/MV transformer is normally supplied by a single feeder. In case of a fault or interruption on one of the supplies, the other can ensure continuity of supply.

   ✓ **Characteristics**:

   - Provides increased security compared to the single-feeder configuration,
   - Allows partial or complete service continuity,
   - More costly than the single-feeder scheme due to the duplication of lines.

✓ **Limitation** :

Redundancy is not always complete: if both antennas come from the same source station, they remain vulnerable to a common upstream failure.



(a) Single Busbar Feed (b) Double Busbar Feed

Figure II.4. Supply of an HV Delivery Substation.

3. Double-Feeder Supply with Double Busbars

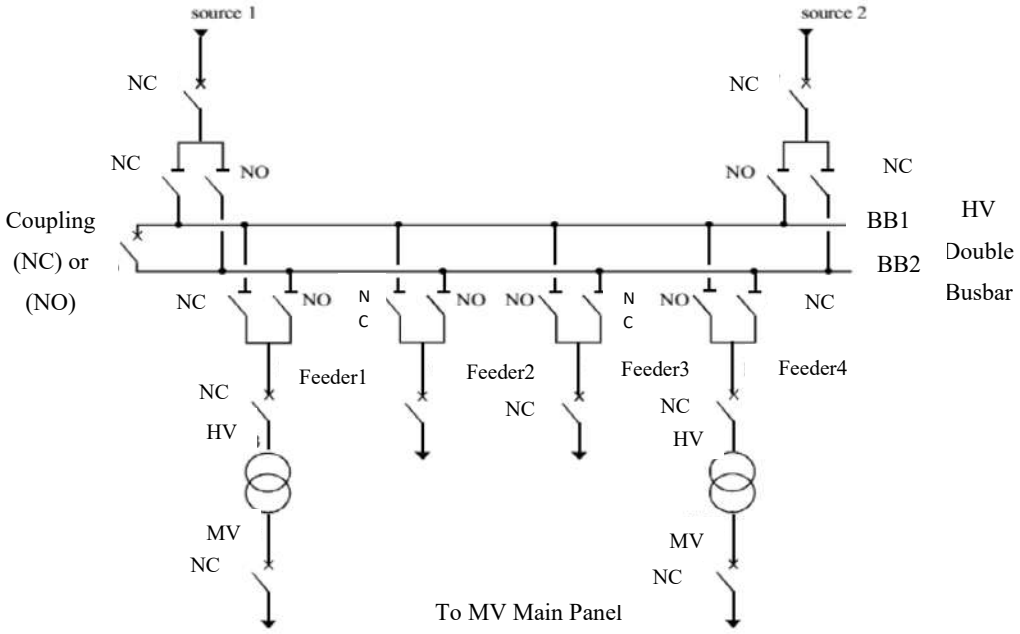


Figure II.5. Double-Feeder, Double-Busbar Supply of a High-Voltage Delivery Substation (HVDS).

Operating Mode

- Normal: Source 1 supplies, for example, busbar BB1 and feeders F1 and F2. Source 2 supplies, for example, busbar BB2 and feeders F3 and F4. The coupling circuit breaker can be kept either closed or open.

- Disturbed: In case of loss of one source, the other source supplies the entire load. In case of a fault on a busbar (or during its maintenance), the coupling circuit breaker is opened, and the other busbar supplies all the feeders..

- **Advantages:**

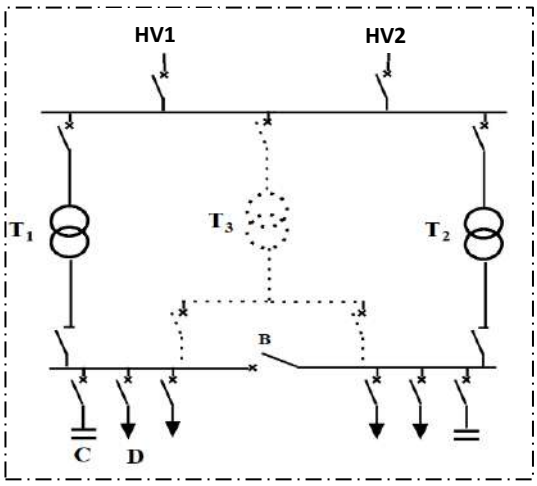
- Supply availability: the presence of two feeders and a double busbar ensure service continuity even in case of loss of one source or maintenance of a busbar.
- Flexibility in assigning sources and loads, as well as for busbar maintenance.
- Interruption-free transfer: it is possible to transfer the load from one busbar to another without interruption, thanks to the temporary coupling of busbars and the coordinated operation of disconnectors and circuit breakers.

- **Disadvantages**:
  - High cost: this architecture incurs a significant additional cost compared to a simpler configuration, such as a single-busbar substation.

II.6.2. HV/MV Primary Substations

The medium-voltage (MV) distribution network begins at the HV/MV primary substation, from which several MV feeders, consisting of conductors and switching devices, originate. These feeders supply medium-voltage loads or public distribution substations (MV/LV).

A primary substation, in its initial configuration, consists of a transformer (T1) supplied by a high-voltage line (HV1). As the load to be served increases, a second transformer (T2) can be added, and in the final phase, a third transformer (T3) is often installed in double connection to enhance operational security. Alongside the addition of the second transformer, a second HV supply (HV2), called the line guarantee, is generally connected to ensure service continuity in case of a fault on the first one (Figure II.6).



B : MV busbar with two half-buses

C : Capacitor Bank

D : MV feeders

HV1, HV2 : HV supplies

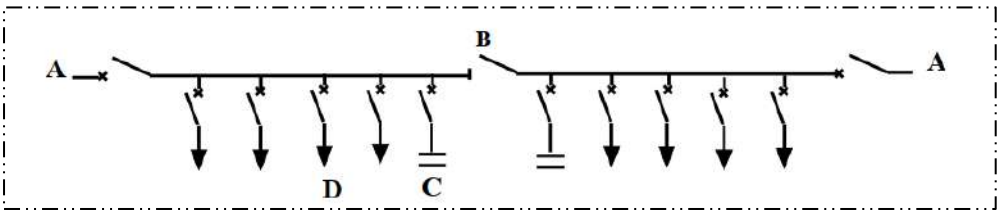
T_1, T_2 : HV/MV Transformers

T3 : troisième transformateur raccordé en double attache

Figure II.6. Diagram of an HV/MV Substation.

The transformers supply an MV switchboard, which constitutes a busbar system organized in sections. Each section consists of approximately ten bays and is divided into two half-sections, connected by a coupling device called a busbar disconnector (Figure II.7). A typical half-section generally includes:

- A transformer feeder.
- Several MV feeders.
- A capacitor bank intended for reactive power compensation.
- Optionally, a shunt circuit breaker.



A : Transformer Incomer C : capacitor bank B Coupling between half-sections D : MV feeders

Figure II.7. Diagram of the Principle of an MV Section.

II.7. MV Networks

The medium-voltage (MV) distribution network generally has a radial, tree-like structure, which can be looped through another half-section or a primary substation to enhance operational security. It consists of a main framework and several branches. Depending on the

load density to be served, this network can be implemented with overhead lines or underground cables.

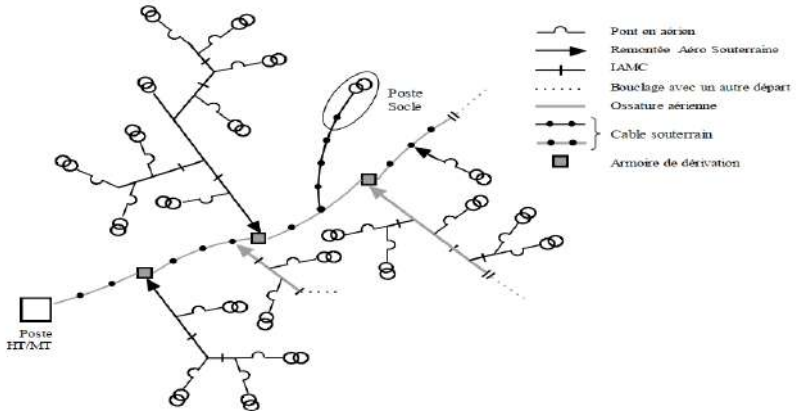


Figure II.8. Structure of an MV Feeder

1. Underground MV Networks

Underground MV networks differ from overhead networks mainly by the longer outage durations in case of a failure (10 to 20 hours for locating and repairing), as well as for scheduled maintenance works. Their reliability depends on the number of supply paths available for the same MV/LV substation:

- A single path (radial, single-feeder structure) is simple and economical but is not recommended for underground networks, as it does not allow service restoration.
- Two-path structures are the most common, with two variants: the double-branch and the main-artery sectioning.
- Multi-path structures, less common, provide superior service quality.

➢ Double-Branch Network

The doubled radial (single-feeder) network is supplied from the busbar of the HV/MV primary substation. Each MV/LV transformer is connected to a working cable (WC) for normal operation and to a backup cable (BC), which is activated in case of a fault via switching devices (Figure II.9). Although this architecture is difficult to operate manually, it can be easily automated and ensures better service continuity for supplying MV/LV substations. However,

its main drawback is the significant cable length required, which increases the cost.

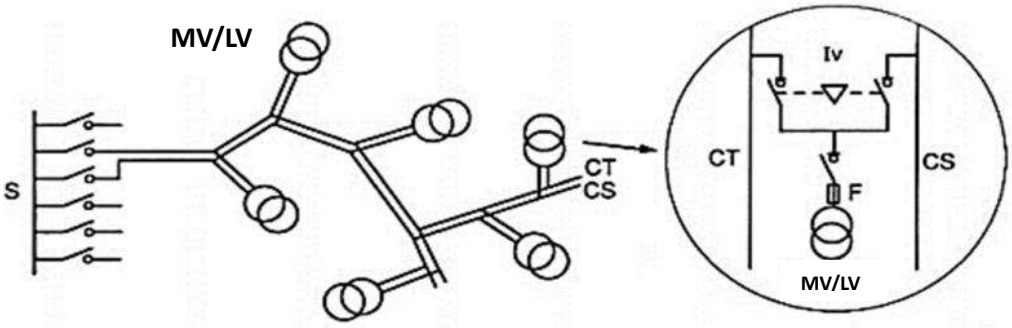


Figure II.9. Double-Branch Network.

- **Multi-Branch Network**

Each HV/MV/LV primary substation has two supply paths; each connected to several cables (between three and six) originating from the HV/MV substation. In case of a fault, the load can be transferred either to a dedicated backup cable or to one of the other cables providing integrated backup (Figure II.10).

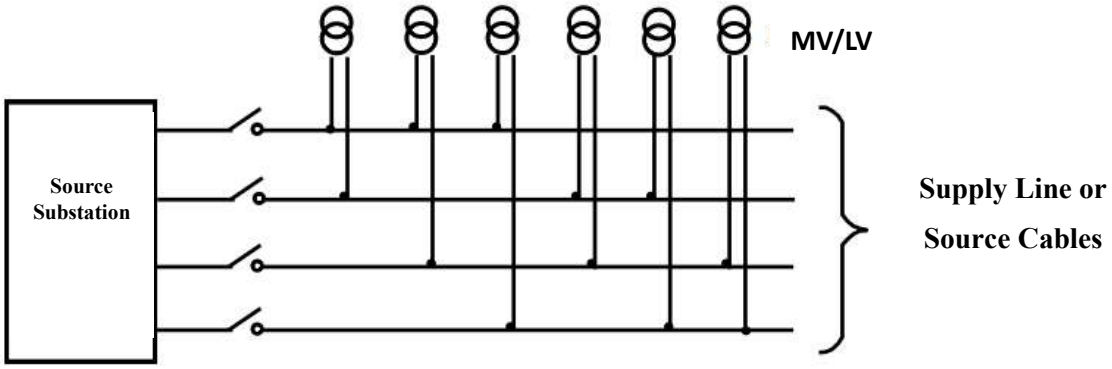


Figure II.10. Multi-Path Supply.

- **Main-Artery Sectioned Network**

In this type of network, HV/MV primary substations are interconnected in pairs by circuits called arteries (Figure II.11). MV/LV substations are directly connected to these arteries, which include one or more switching devices, kept open or closed according to operational requirements to maintain a radial structure. In some cases, the connection between HV/MV substations can be reinforced with a backup cable.

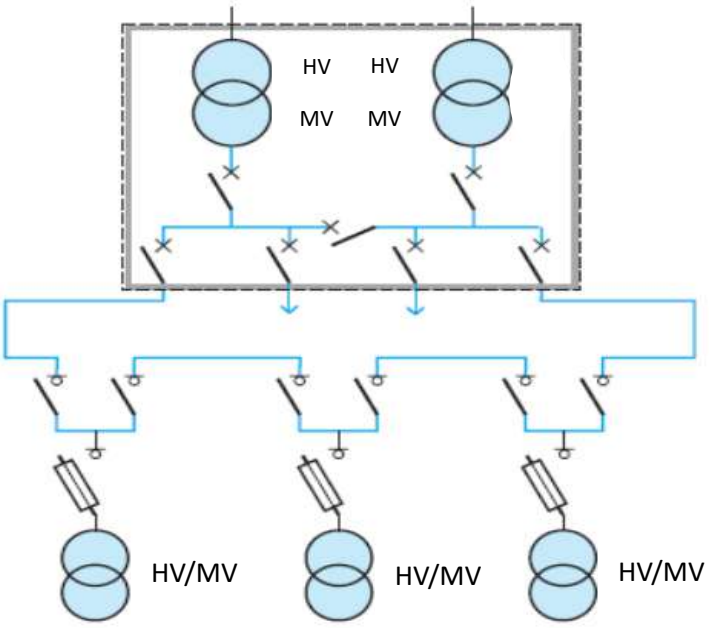


Figure II.11.MV Network with Main-Artery Sectioning.

The main-artery sectioned structure has several variants, known as spindle, branch, and loop (or daisy-petal). Figures II.12 illustrate these three configurations.

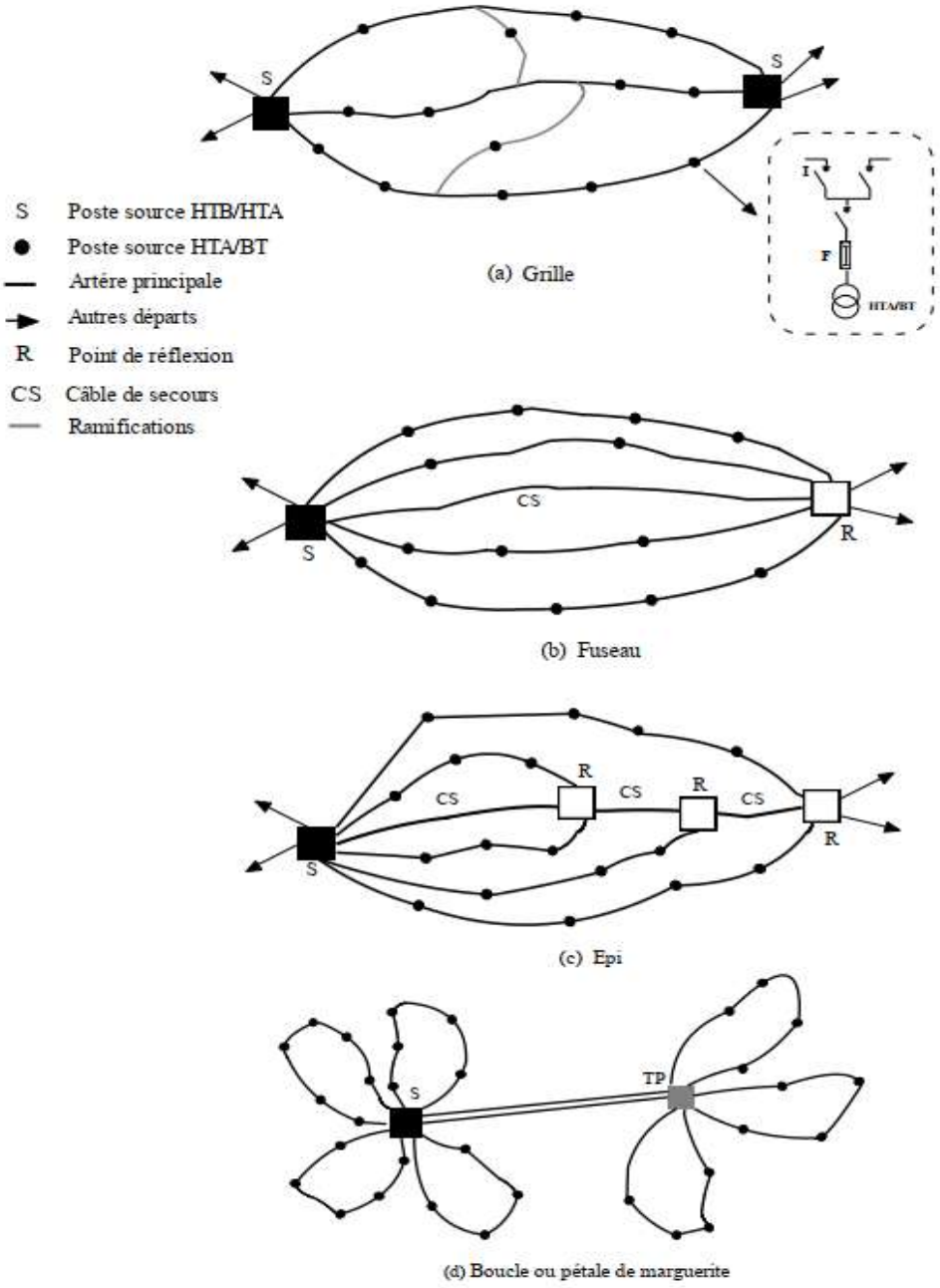


Figure II.12. Réseaux en coupure d'artère

II.7.1. MV/LV or MV Substations

MV/LV substations serve as the interface between medium-voltage (MV) and low-voltage (LV) networks. Their primary role is the transformation of MV power into LV, to which may be added, depending on requirements, an MV operational function (switching point) and an LV distribution function. One of their essential characteristics is the rated power, which can range

from a few kVA to several MVA depending on the context and the loads to be served. Accordingly, different types of MV/LV substations are distinguished, exhibiting increasing levels of complexity.

A. Pole-Mounted Substation, called H61

The pole-mounted aerial substation is the simplest type of MV/LV substation, primarily used in overhead networks. Designed as an extension of the line, it combines on the same pole the MV incomer (in single-branch configuration without a switching device), a visible transformer, and an LV outlet equipped with a circuit breaker located mid-pole (Figure II.13). This circuit breaker provides universal overcurrent protection.

Inexpensive and easy to install, this type of substation has greatly contributed to the rapid electrification of rural areas, thanks to its simplicity and operational flexibility. The standard transformer ratings used are 63, 100, and 160 Kva.



Figure II.13. Pole-Mounted Substation.

B. Simplified Ground-Mounted Substation under Enclosure

The prefabricated substation is intended exclusively for MV overhead networks. Compact (approximately 3 m² with a height of 1.50 m) and simplified, it delivers higher power than the H61 type, with standard ratings of 160, 250, and 400 kVA, while remaining economical.The connection to the MV network is made via an overhead-underground drop with a dry cable, without a switching device, and the connection to the transformer is ensured by a plug-in connector.LV distribution is carried out through an assembly equipped with a switching device and can include up to four feeders protected by fuses.

This type of modern substation has gradually replaced the old traditional masonry cabins, which were considered costly and are now obsolete



Figure II.14. Simplified Ground-Mounted Substation under Enclosure.

C. Urban-Type Substation with Underground Connection

They include MV equipment generally consisting of two MV incomers (MV1, MV2) with switchgear in protected design, and a transformer protection cell with a fuse and, optionally, a switch.The substation is usually designed for a single transformer, but in some cases, several transformers may be installed to handle significant peak loads. Standard transformer ratings are 250, 400, 630, and 1000 kVA.



Figure II.15. Urban-Type Substation with Underground Connection.

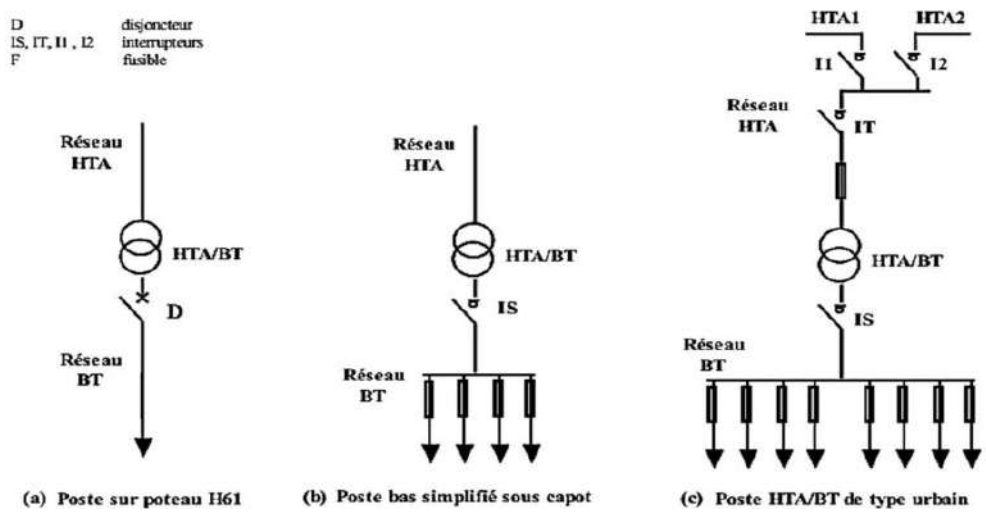


Figure II.16. MV/LV Substation Structures.

II.8. Low-Voltage (LV) Networks

There are different supply modes for low-voltage (LV) panels, which can be fed by a single source or multiple sources. Depending on requirements in terms of service continuity, operational safety, and flexibility, these sources may come from the public network, generators, or decentralized production units such as photovoltaic, wind, or hybrid systems.

II.8.1. Supply of LV panels with a Single Power Source

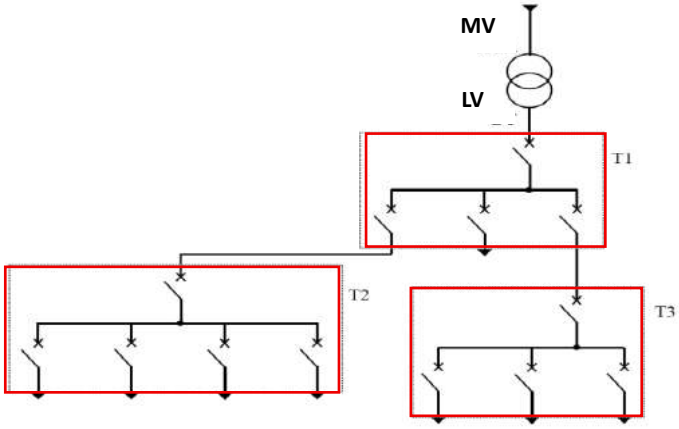


Figure II.17. Supply of LV panels with a Single Power Source.

tables T1, T2, and T3 are fed by a single power source, which defines a tree-like radial network. In this distribution mode, each panel has only one supply path.

In the event of a failure or interruption on the main source, the affected panel is immediately taken out of service and remains unavailable until the repair operation is completed.

II.8.2. Supply of LV panels by Dual Power Sources without Coupling

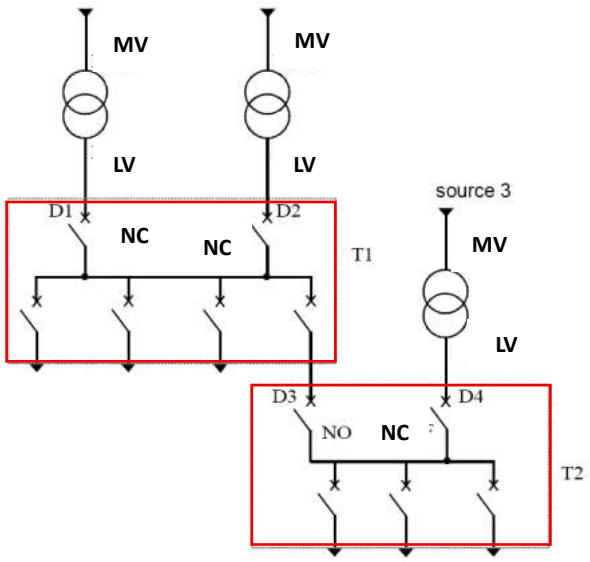


Figure II.18. Supply of LV panels by Dual Power Sources without Coupling.

Table T1, shown in Figure II.18, is supplied by a dual power source without coupling via two MV/LV transformers:

- Both sources feed T1 in parallel.
- Under normal operation, both circuit breakers are closed (D1 and D2)

Table T2 is supplied by a dual power source without coupling via one MV/LV transformer and a feeder from another LV table:

- One source feeds table T2, while the second provides backup;
- Under normal operation, only one circuit breaker is closed (D3 or D4).

II.8.3. Supply of LV panels by Dual Power Sources with Coupling

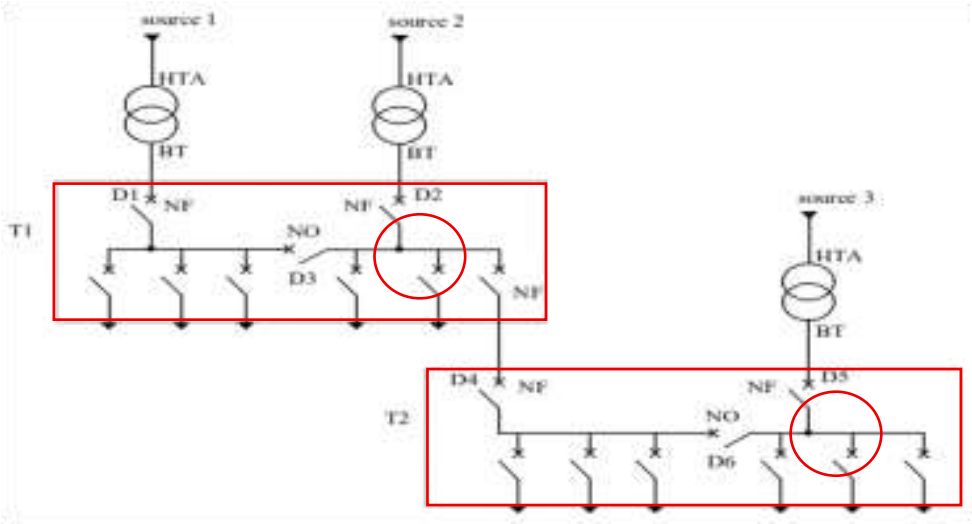


Figure II.19. Supply of LV Panels by Dual Power Sources with Coupling.

Table T1

Table T1 is powered by a dual power supply with coupling, provided by two MV/LV transformers.

Normal operation: the D3 coupling circuit breaker is open. Each transformer handles a separate section of the panel, allowing for balanced load distribution.

In the event of a power source failure: circuit breaker D3 closes automatically, and the remaining transformer in service takes over the power supply for the entire T1 panel.

Table T2

The T2 panel also has a dual power supply with coupling, but with a different configuration:

- The first source is a dedicated MV/LV transformer.
- The second source comes from a LV outlet from another panel.
- Normal operation: the D6 coupling circuit breaker is open, and each source supplies a separate section of the panel.
- In the event of loss of one source: the D6 circuit breaker closes, allowing the remaining source to supply the entire T2 panel on its own.

Chapter III

Power line modeling

III.1. Introduction

A high-voltage transmission line, whether overhead or underground, is used to transfer electrical energy from generators to loads. Each line is characterized by its specific parameters: resistance, inductance, and capacitance. According to the principle of power conservation, the use of high voltage offers two main advantages:

- Increasing the voltage reduces the current,
- Decreasing the current allows the use of conductors with smaller cross-sections.

To increase the power transfer capability, transmission lines may use multiple conductors per phase, grouped in bundles.

III.2. Components of a Transmission Line

A transmission line is characterized by four electrical parameters: resistance (R), inductance (L), capacitance (C), and conductance (G).

- The series longitudinal parameters are the resistance and inductance (R, L), which define the line reactance.$X' = j\omega L'$.

$$Z'_{\text{longitudinale}} = R' + jX' \quad [\Omega/m] \qquad \textbf{III.1}$$

- The shunt transverse parameters are the capacitance and conductance (C, G). The effective transverse admittance is composed of the line susceptance:

$$Y' = j\omega C' \qquad [S/m] \qquad \textbf{III.2}$$

The conductance represents the leakage currents between conductors or between conductors and the ground, mainly through the insulation of cables and along the insulators. However, in the case of overhead lines, these leakages are very small and conductance is generally neglected. The impedances of a transmission line are uniformly distributed along its entire length.

Thus, the line can be modeled as a succession of identical sections containing R, L, and C. Each section corresponds to a segment of a given length (for example, 1 km), where r, X_L and X_C respectively represent the resistance, inductive reactance, and capacitive reactance associated with this portion of the line (Figure III.1).

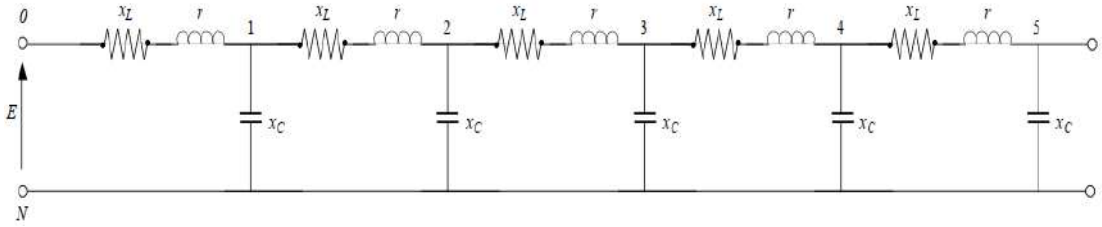


Figure III.1. Representation of a power line by section.

The line model can be simplified by adding the elementary resistances to obtain a total resistance R. Similarly, the sum of the inductances gives a total inductive reactance X_L and the sum of the capacitances provides a total capacitive reactance X_C.

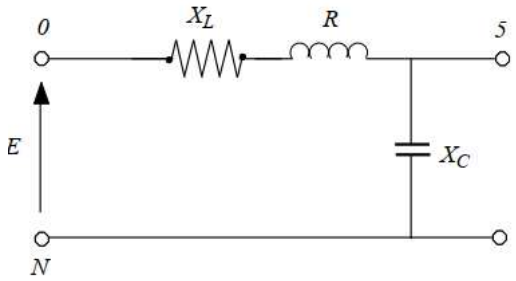


Figure III.2. Equivalent circuit of an electrical line.

III.3. Longitudinal characteristics

III.3.1. Resistance

The resistance of conductors is the main cause of Joule effect losses. Three factors determine the resistance value of a conductor at a given temperature:

- **The nature of the material**, characterized by its resistivity.
- **The cross-sectional area of the conductor**, which is inversely proportional to its resistance.
- **The length of the conductor**, which is directly proportional to its resistance: the longer the conductor, the higher its resistance.

When an electric current flows through a homogeneous cylindrical conductor, it is uniformly distributed over the entire cross-section of the conductor (Figure III.3). The resistance encountered by this direct current, taking into account the three factors previously mentioned, is given by the following formula:

$$R = \frac{\rho \times L}{S} \quad [\Omega] \qquad \textbf{III. 3}$$

Where: ρ : is the resistivity of the material (Ω.m), L : is the length (m) et S : is the cross-sectional area of the conductor (m^2).



Figure III.3. Uniform distribution of direct current inside a homogeneous cylindrical conductor.

✓ **Effect of temperature**

Resistivity depends on temperature T and is expressed as follows:

$$\rho(T) = \rho(T_0) + [1 + \alpha(T - T_0)] \qquad \textbf{III.4}$$

Where :

- ρ (T_0) is the resistivity of the conductor at 20 °C [Ωm]
- α : is the temperature coefficient [$°C^{-1}$] (α=0.004 $°C^{-1}$ for the Cu et Al).

The resistance of the conductor increases with rising temperature. Since the value of ρ is given at a specific temperature and the line operates at a higher temperature, the actual resistance (equation III.5) is higher than the value obtained in equation (III.3). For small temperature variations, the resistance increases linearly as the temperature rises (Figure III.4).

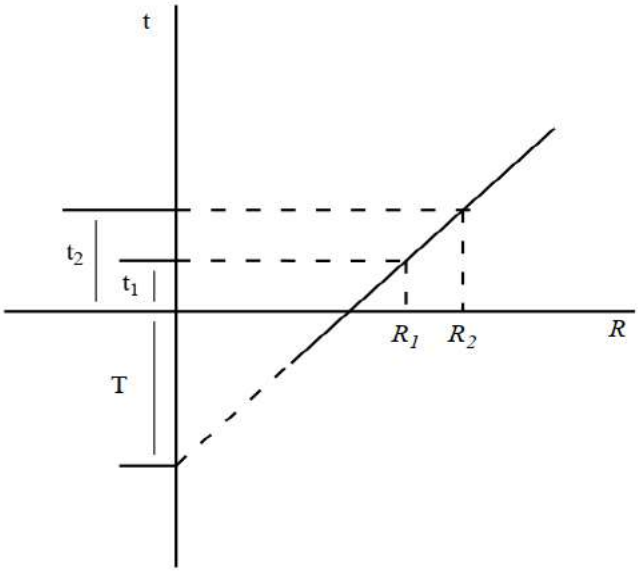


Figure III.4. Variation in the resistance of a conductor as a function of temperature.

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1} \qquad \textbf{III. 5}$$

Where: R_1 et R_2 represent the conductor resistances at temperatures t1 and t2 (expressed in °C), respectively, while T denotes the characteristic constant determined from the graph. The values of the constant T in degrees Celsius are as follows:

Temperature in °C	Conductor type
234.5	For annealed copper with 100% conductivity
241	For drawn copper with 97.3% conductivity
228	For aluminum (and its alloys) with 61% conductivity

Note: The resistance of a conductor depends not only on its resistivity, length, and cross-sectional area, but also on frequency (skin effect), wire winding, and temperature, which tends to increase it.

✓ **Influence of frequency: skin effect (or Kelvin effect)**

In addition to the influence of materials and temperature, the resistance of a conductor also varies with frequency. In alternating current, the current density is not uniformly distributed across the cross-section of the conductor. As the frequency increases, the current density gradually shifts toward the periphery of the conductor. This phenomenon, known as the **skin effect** (sometimes referred to as the Kelvin effect), reduces the effective conducting area, which in turn increases the electrical resistance and modifies the inductance of the conductor.

The Kelvin factor is calculated using the following expression:

$$q = \pi \times d \sqrt{\frac{2.f10^{-5}}{\rho}} \qquad \textbf{III. 6}$$

Where :

- d : conductor diameter in cm.
- f : network frequency in Hz.
- ρ : metal resistivity (Ω. mm²/km).

III.3.2. Longitudinal reactance (inductance)

When a current flows through a conductor, it generates a magnetic field around it, composed of concentric flux lines. If the current varies, the associated magnetic flux also changes, thereby inducing a voltage in the circuit.

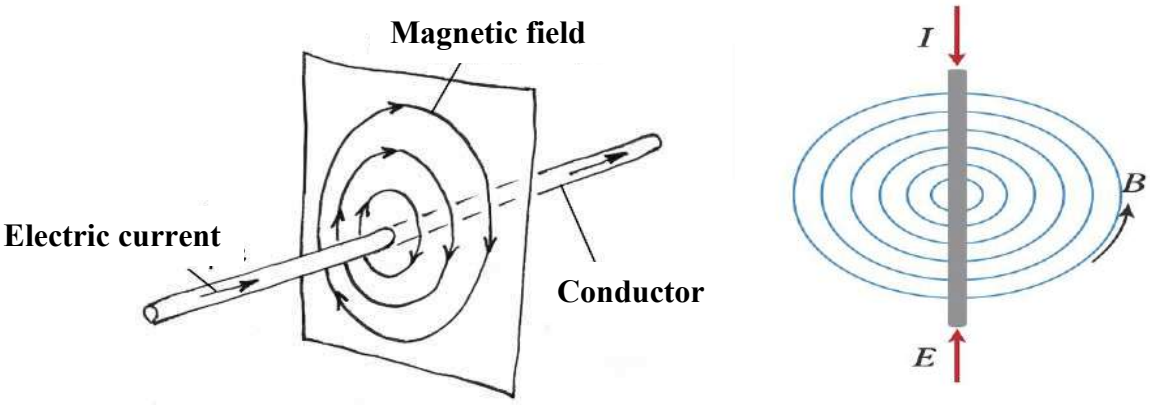


Figure III.5. Magnetic field around a conductor carrying an electric current.

By definition, for a non-magnetic material, the inductance L is defined as the ratio between the total magnetic flux linked to the conductor and the current I that produces it:

$$L = \frac{\lambda}{I} \qquad \textbf{III. 7}$$

Where :

λ : is the induction flux linkage with $\lambda = N\varphi$ *en [Wb / m].*

I : is the current flowing through the conductor.

III.3.2.A. Conductance of a single conductor

We consider an infinitely long cylindrical conductor with radius r and a uniform circular cross-section (Figure III.6), carrying a total current I. Inside this conductor, at a distance x from the center, the current enclosed by a circle of radius xxx is denoted by I_x.

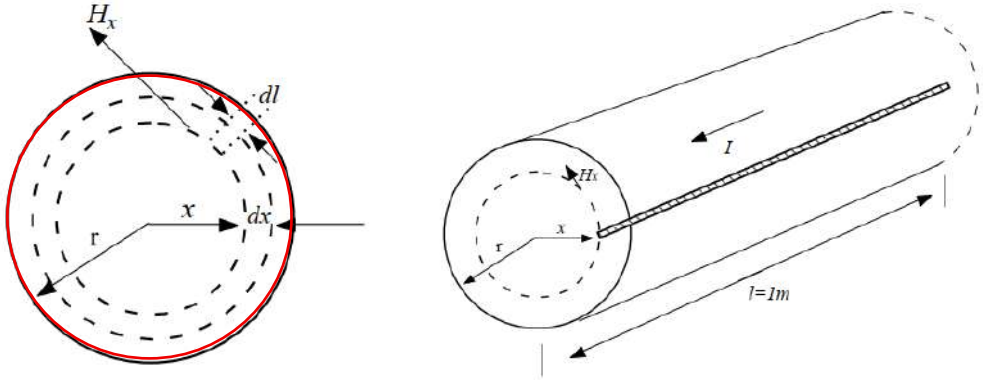


Figure III.6. Magnetic induction outside a long circular conductor.

Applying Ampere's law along a circular path of radius x, where the magnetic field H is constant:

$$\int_0^{2\pi x} H.dl = I_x \qquad \textbf{III. 8}$$

$$H_x.2\pi x = I_x \qquad \textbf{III. 9}$$

$$H_x = \frac{I_x}{2\pi x} \quad (At/m) \qquad \textbf{III.10}$$

The inductance of the conductor can be defined as the sum of the contributions of the internal and external flux links to the conductor.

✓ **Internal inductance**

To determine the internal inductance, we consider the magnetic field produced inside the conductor, at a distance x from the axis, over a segment of length l.

Assuming a uniform distribution of the current over the entire cross-section of the conductor, and neglecting the skin effect, the current density can be expressed as:

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \qquad \textbf{III.11}$$

By substituting the value of *Ix* into equation (III.10), we obtain:

$$H_x = \frac{I}{2\pi r^2} x \qquad \textbf{III.12}$$

For a non-magnetic conductor, characterized by a constant permeability μ_0, the magnetic flux density at a distance x from the axis is expressed as: $B_x = \mu_0 H_x$

$$B_x = \frac{\mu_0 I}{2\pi r^2} x \qquad \textbf{III.13}$$

Where μ_0 represents the magnetic permeability of a vacuum (or air), whose value is:$\mu_0 = 4\pi \times 10^{-7}$ H/m

The differential magnetic flux φ through an elementary ring with radius x, thickness dx, and unit length (l=1m) is given by:

$$d\varphi = B_x . dS = B_x . dx . 1 \qquad \textbf{III.14}$$

Assuming that the current density is uniform:

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \quad \rightarrow \quad I_x = \frac{x^2}{r^2} I \qquad \textbf{III.15}$$

The differential flux φ is only related to the part of the conductor between the axis and the radius x. Consequently, only a fraction of the total current I is actually related to this flux.

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx \qquad \textbf{III.16}$$

The total flux linkage is obtained by integrating $d\lambda_x$ from 0 to *r* .

$$\lambda_{int} = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx = \frac{\mu_0 I}{8\pi} \left(Wb/m\right) \qquad \textbf{III.17}$$

From equation (III.7), the inductance due to the internal flux linkage is:

$$L_{int} = \frac{\lambda_{int}}{I} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \quad \left(H/m\right) \qquad \textbf{III.18}$$

✓ **Inductance externe**

Let us consider the external magnetic field H(x) f a long cylindrical conductor of radius r. For a point located at a distance x > r from the axis (Figure III.7), since the circle of radius x encloses the entire current, we have, $I_x = I$

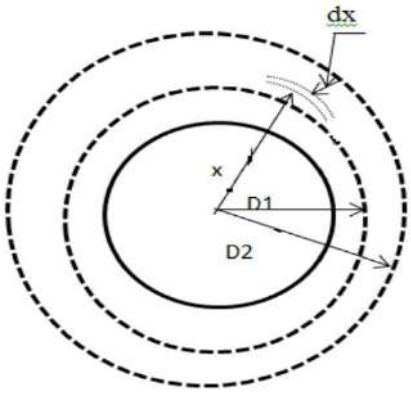


Figure III.7. Solid circular section conductor.

Replacing the current Ix with I in equation (III.10), the flux density at radius x becomes:

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x} \qquad \textbf{III.19}$$

Since the total current I is surrounded by the flux outside the conductor, the flux linkage $d\lambda_x$ is numerically equal to the flux $d\,\Phi x$. As in the previous case:

$$d\lambda_x = d\varphi_x = B_x.dx.1 = \frac{\mu_0 I}{2\pi x}dx \qquad \textbf{III.20}$$

The external flux linkage between two points D1 and D2 is given by:

$$\lambda_x = \frac{\mu_0 I}{2\pi}\int_{D2}^{D1}\frac{1}{x}dx = 2.10^{-7} I\, ln\frac{D2}{D1} \qquad \textbf{III.21}$$

The inductance between two points external to a conductor is:

$$L_{ext} = 2.10^{-7}\, ln\frac{D2}{D1} \qquad \textbf{III.22}$$

III.3.2.B. Single-phase line inductance

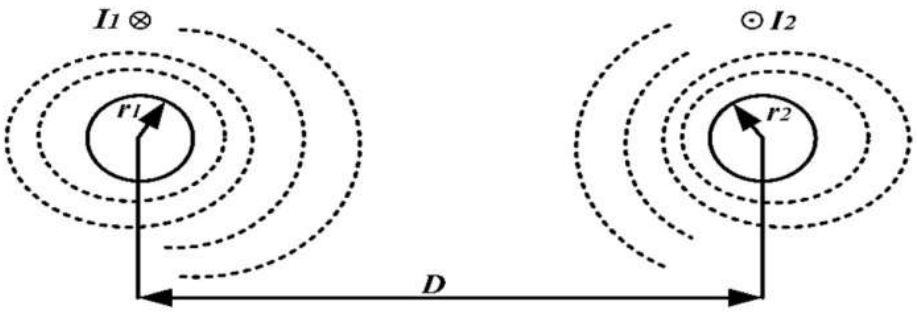


Figure III.8. Single-phase two-wire line.

The total inductance of conductor 1 is therefore:

$$L_1 = \frac{1}{2}.10^{-7} + 2.10^{-7}\, ln\frac{D}{r_1}\ \left(^{H}/_{m}\right) \qquad \textbf{III.23}$$

Equation (III.23) can also be expressed in the following form:

$$L_1 = 2.10^{-7}\left[\frac{1}{4} + ln\frac{D}{r_1}\right]$$

$$= 2.10^{-7}\left[ln\, e^{1/4} + ln\frac{1}{r_1} + ln\frac{D}{1}\right] \qquad \textbf{III.24}$$

$$= 2.10^{-7}\left[ln\frac{1}{r_1 e^{-\frac{1}{4}}} + ln\frac{D}{1}\right]$$

Where : $r_1' = r_1 e^{-\frac{1}{4}}$

So the inductance of conductor 1 is:

$$L_1 = 2.10^{-7}\left[ln\frac{1}{\mathrm{r}_1'} + ln\frac{D}{1}\right]\left(^{H}/_{m}\right) \qquad \textbf{III.25}$$

The same applies to the second driver:

$$L_2 = 2.10^{-7}\left[ln\frac{1}{\mathrm{r}_2'} + ln\frac{D}{1}\right]\left(^{H}/_{m}\right) \qquad \textbf{III.26}$$

If the two conductors are identical $\boldsymbol{r_1' = r_2' = r'}$ et $L_1 = L_2 = L$ The inductance per phase per meter of length is given by:

$$L = 2.10^{-7}\left[ln\frac{1}{\mathrm{r}'} + ln\frac{D}{1}\right]\left(^{H}/_{m}\right) \qquad \textbf{III.27}$$

$$L = 2.10^{-7}\left[ln\frac{D}{\mathrm{r}'}\right]\left(^{H}/_{m}\right)$$

The term $r' = r\, e^{-1/4}$ is known mathematically as the Geometric Mean Radius (GMR) of a circle with radius r, and is designated by D_S. $D_S = r\, e^{-1/4}$

So inductance can be written as:

$$L = 2.10^{-7} ln\frac{D}{D_s}\left(^{H}/_{m}\right) \qquad \textbf{III.28}$$

$$L = 0{,}2\, ln\frac{D}{D_s}\left(^{mH}/_{Km}\right)$$

III.3.2.C. Inductance of a symmetrical three-phase line

Consider a three-phase line consisting of three identical conductors of radius r, arranged symmetrically in a triangle (Figure III.9). The line carries a balanced system of currents (I_a ,I_b, I_c) verifying $I_a+I_b+I_c=0$.

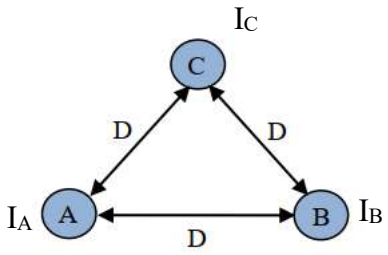


Figure III.9. Three-phase line with balanced spacing.

The total flux linkage of phase conductor a is:

$$\lambda_a = 2.10^{-7}\left[I_a\, ln\frac{1}{r'} + I_b\, ln\frac{1}{D} + I_c\, ln\frac{1}{D}\right] \qquad \textbf{III.29}$$

By substituting : $-I_a = I_b + I_c$

$$\lambda_a = 2.10^{-7}\left[I_a\, ln\frac{1}{r'} - I_a\, ln\frac{1}{D}\right] = 2.10^{-7}\left[I_a\, ln\frac{D}{r'}\right] \qquad \textbf{III.30}$$

Because of symmetry $\lambda_a = \lambda_b = \lambda_c$,and the three inductors are symmetrical. Therefore:

$$L = 2.10^{-7} ln\frac{D}{D_s}\left({}^{H}/_{m}\right) \qquad \textbf{III.31}$$

$$L = 0{,}2\, ln\frac{D}{D_s}\left({}^{mH}/_{Km}\right)$$

III.3.2.D. Inductance of an unbalanced three-phase line (transposed line)

Three-phase lines are not always arranged in a perfectly symmetrical manner, which leads to an imbalance of inductances between the different phases. To compensate for this asymmetry, phase transposition is carried out (see Figure III.10), which helps restore balance. In this case, the concept of average distance between phases is introduced, defining a more regular equivalent configuration.

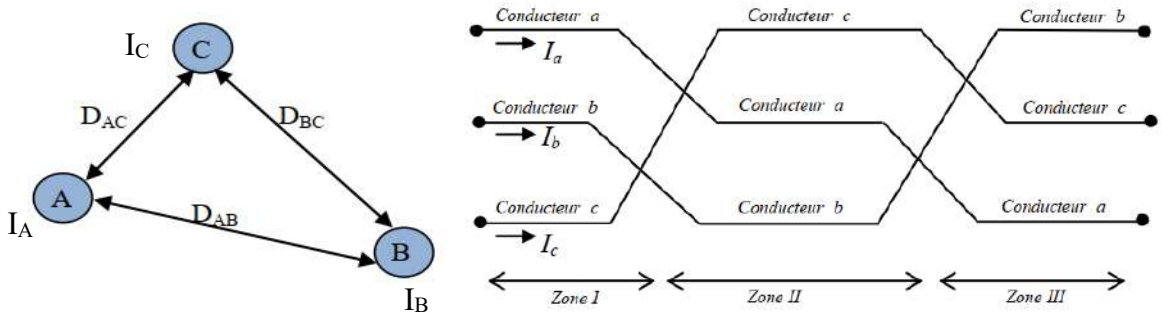


Figure III.10. Transposition of conductors in a three-phase line.

The flux links associated with each conductor are expressed by:

$$\lambda_a = 2.10^{-7}\left[I_a\, ln\frac{1}{r'} + I_b\, ln\frac{1}{D_{12}} + I_c\, ln\frac{1}{D_{13}}\right]$$

$$\lambda_b = 2.10^{-7}\left[I_a ln\frac{1}{D_{12}} + I_b\, ln\frac{1}{r'} + I_c\, ln\frac{1}{D_{23}}\right] \quad \textbf{III.32}$$

$$\lambda_c = 2.10^{-7}\left[I_a ln\frac{1}{D_{13}} + I_b\, ln\frac{1}{D_{23}} + I_c\, ln\frac{1}{r'}\right]$$

For a balanced three-phase system, and considering Ia as a reference, the currents are expressed as follows:

$I_b = I_a\angle 240° = a^2 I_a$

$I_c = I_a\angle 120° = aI_a$

With : $a = 1\angle 120°\ et\ a^2 = 1\angle 240°$.

So :

$$L_a = \frac{\lambda_a}{I_a} = 2.10^{-7}\left[ln\frac{1}{r'} + a^2 ln\frac{1}{D_{12}} + a\, ln\frac{1}{D_{13}}\right]$$

$$L_b = \frac{\lambda_b}{I_b} = 2.10^{-7}\left[a\, ln\frac{1}{D_{12}} + ln\frac{1}{r'} + a^2\, ln\frac{1}{D_{23}}\right] \quad \textbf{III.33}$$

$$L_c = \frac{\lambda_c}{I_c} = 2.10^{-7}\left[a^2\, ln\frac{1}{D_{13}} + a\, ln\frac{1}{D_{23}} + ln\frac{1}{r'}\right]$$

In the case of a transposed line, the inductance per phase is given by:

$$L = \frac{L_a + L_b + L_c}{3}$$

And let's also note that: $a^2 + a = -1$

$$L = \frac{2.10^{-7}}{3}\left[3\, ln\frac{1}{r'} - ln\frac{1}{D_{12}} - ln\frac{1}{D_{23}} - ln\frac{1}{D_{13}}\right]$$

$$L = 2.10^{-7}\left[ln\frac{1}{r'} - ln\frac{1}{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}\right] \quad \textbf{III.34}$$

$$L = 2.10^{-7}\, ln\frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{r'}\ (H/m)$$

Where the inductance per phase per kilometer is given by:

$$L = 0.2\, ln\frac{DMG}{D_s}\ \left({}^{mH}/_{Km}\right) \quad \textbf{III.35}$$

Où : $DMG = \sqrt[3]{(D_{12}D_{23}D_{13}}$

Where GMD (Geometric Mean Distance) is the equivalent spacing between conductors. DSD_SDS is the Geometric Mean Radius (GMR) for a solid round conductor, given by:

$$D_s = r_1' = r_1 e^{-\frac{1}{4}}$$

III.3.2.e Inductance of composite conductors

Let us consider a single-phase line consisting of two composite conductors xxx and yyy, as illustrated in Figure III.11. It is assumed that the current is uniformly distributed among the sub-conductors. The current flowing through each strand is given by: $\frac{1}{n}$ for x and $-\frac{1}{m}$ for y.

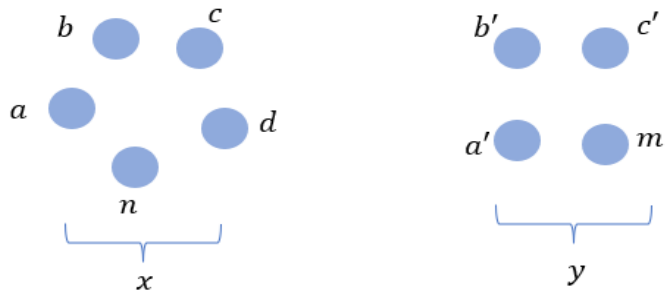


Figure III.10. Single-phase line with two composite conductors.

$$\lambda_a = 2.10^{-7}\frac{I}{n}\left[ln\frac{1}{r_{x'}} + ln\frac{1}{D_{ab}} + ln\frac{1}{D_{ac}} + \cdots\cdots ln\frac{1}{D_{an}}\right] - 2.10^{-7}\frac{I}{m}\left[ln\frac{1}{D_{aa'}} + ln\frac{1}{D_{ab'}} + ln\frac{1}{D_{ac'}} + \cdots\cdots ln\frac{1}{D_{am}}\right] \quad \textbf{III.36}$$

$$\lambda_a = 2.10^{-7} I\, ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}..D_{am}}}{\sqrt[n]{r_x' D_{ab}D_{ac}..D_{an}}} \quad \textbf{III.37}$$

The sub-conductor inductance is:

$$L_a = \frac{\lambda_a}{I/n} = 2n.10^{-7} I\, ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}..D_{am}}}{\sqrt[n]{r_x' D_{ab}D_{ac}..D_{an}}} \quad \textbf{III.38}$$

Similarly, for the other subconductors belonging to conductor x, we obtain:

$$L_n = \frac{\lambda_n}{I/n} = 2n.10^{-7} I\, ln\frac{\sqrt[m]{D_{na'}D_{an}D_{nc'}..D_{nm}}}{\sqrt[n]{D_{na}D_{nb}D_{nc}..r_n'}} \quad \textbf{III.39}$$

The average inductance of each of the sub-conductors in group x is given by:

$$L_{moy} = \frac{L_a + L_b + L_c \ldots L_n}{n} \quad \textbf{III.40}$$

Since the sub-conductors of conductor x are electrically parallel:

$$L_x = \frac{L_a + L_b + L_c \ldots L_n}{n^2} \quad \textbf{III.41}$$

By substituting the values of $L_a + L_b + L_c \ldots L_n$ from equation (III.41), we obtain:

$$L_x = 2.10^{-7} ln\frac{DMG}{RMG_x}\ (H/m) \quad \textbf{III.42}$$

Where :

$$DMG = \sqrt[mn]{(D_{aa'}D_{ab'}D_{ac'}..D_{am})(D_{na'}D_{nb'}D_{nc'}..D_{nm})} \quad \textbf{III.43}$$

And:

$$RMG_x = \sqrt[n^2]{(D_{aa}D_{ab}..D_{an})(D_{na}D_{nb}..D_{nn})} \qquad \textbf{III.44}$$

With :

$$D_{aa} = D_{bb} = \cdots D_{nn} = r'_x = r_x e^{-0;25}$$

Similarly, the inductance of the conductor is:

$$L_y = 2.10^{-7} ln \frac{DMG}{RMG_y} \ (H/m) \qquad \textbf{III.45}$$

Or the DMG is the same as that of the driver x equation (III.43),RMG_y is different :

$$RMG_y = \sqrt[m^2]{(D_{a'a'}D_{a'b'}..D_{a'm})(D_{ma'}D_{mb'}..D_{mm})} \qquad \textbf{III.46}$$

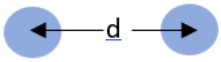
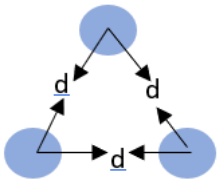
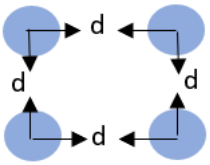
Avec :

$$D_{a'a'} = D_{b'b'} = \cdots D_{mm} = r'_y = r_y e^{-0;25}$$

III.3.2.f Geometric Mean Radius (GMR) of Bundled Conductors

A phase conductor may consist of a bundle of two or more sub-conductors of the same diameter, arranged symmetrically in relation to each other.

Tableau III.1.RMG des conducteurs en faisceaux

Disposition	Formule
	$D_s^b = \sqrt[4]{(D_s \cdot d)^2} = \sqrt{D_s \cdot d}$
	$D_s^b = \sqrt[9]{(D_s \cdot d \cdot d)^3} = \sqrt[3]{D_s \cdot d^2}$
	$D_s^b = \sqrt[16]{(D_s \cdot d \cdot d \cdot d \cdot 2^{1/2})^4}$ $= 1{,}09 \sqrt[4]{D_s \cdot d^3}$

III.3.2.g Inductance d'une ligne triphasée à double circuit

Consider a double-circuit three-phase line consisting of two identical three-phase circuits.. Each circuit operates in parallel with the conductors.$a_1 - a_2, b_1 - b_2$ *and* $c_1 - c_2$. Due to the geometric differences between the conductors, the voltage drop caused by the line inductance

may become unbalanced. To restore balance, each phase conductor must be transposed within its group and kept in parallel with respect to the three-phase line. Here, we consider a three-phase double-circuit line with the relative position of the phases $a_1b_1c_1 - a_2b_2c_2$, as illustrated in Figure III.11.

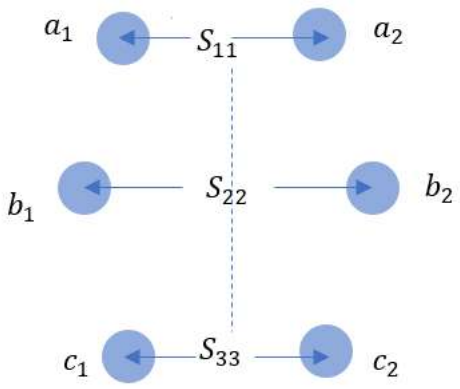


Figure III.11. Transposed dual-circuit line..

The GMD method can be used to determine the inductance per phase. For this purpose, identical phases are grouped together, and then equation (III.43) is applied to calculate the GMD between each group of phases.

$$D_{AB} = \sqrt[4]{D_{a1b1}.D_{a1b2}.D_{a2b1}.D_{a2b2}}$$

$$D_{BC} = \sqrt[4]{D_{b1c1}.D_{b1c2}.D_{b2c1}.D_{b2c2}} \qquad \textbf{III.47}$$

$$D_{AC} = \sqrt[4]{D_{a1c1}.D_{a1c2}.D_{a2c1}.D_{a2c2}}$$

DMG equivalent per phase is::

$$DMG = \sqrt[3]{D_{AB}D_{BC}D_{AC}} \qquad \textbf{III.48}$$

Similarly, from equation (III.44), the GMR of each phase group is:

$$D_{SA} = \sqrt[4]{\left(D_s^b D_{a1a2}\right)^2} = \sqrt{D_s^b D_{a1a2}}$$

$$D_{SB} = \sqrt[4]{\left(D_s^b D_{b1b2}\right)^2} = \sqrt{D_s^b D_{b1b2}} \qquad \textbf{III.49}$$

$$D_{SC} = \sqrt[4]{\left(D_s^b D_{c1c2}\right)^2} = \sqrt{D_s^b D_{c1c2}}$$

The equivalent Geometric Mean Radius (GMR) for calculating the inductance per phase to neutral is:

$$RMG_L = \sqrt[3]{D_{SA}D_{SB}D_{SC}} \qquad \textbf{III.50}$$

The inductance per phase in millihenries per kilometer is:

$$L = 0.2\ ln\frac{DMG}{RMG_L}\ \left({}^{mH}/_{Km}\right) \qquad \textbf{III. 51}$$

III.3.3. Line capacity

III.3.3.a Capacity of a single-phase line

The capacitance of a transmission line is defined as the ratio between the charge accumulated on the conductors and the applied voltage, reflecting the ability of the line to store electrical energy in the form of an electrostatic field. It is given by:

$$C = {}^{q}/_{V} \qquad \textbf{III. 52}$$

Where q is the stored charge and V is the applied voltage.

Consider a long, circular conductor of radius r carrying a linear charge q (in coulombs per meter) as shown in Figure III.12.

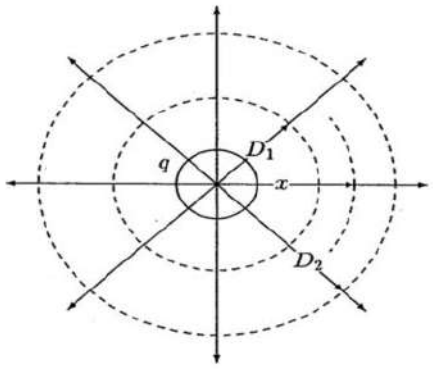


Figure III.12. Electric field around a long round conductor.

Based on Gauss's theorem, the electric flux density passing through a cylindrical surface with radius x and length 1 m is:

$$D = {}^{q}/_{A}$$

$$= \frac{q}{2\pi x(1)} \qquad \textbf{III. 53}$$

The intensity of the electric field E is defined as the ratio between the electric flux density and the permittivity of the medium.

$$E = {}^{D}/_{\varepsilon_0} = \frac{q}{2\pi x\varepsilon_0} \qquad \textbf{III. 54}$$

Where: ε_0 is the permittivity of free space (8.85×10^{-12} F/m) ;

Subsequently, the potential difference between cylinders D1 and D2 is:

$$V_{12} = \int_{D1}^{D2} Edx = \int_{D1}^{D2} \frac{q}{2\pi\varepsilon_0 x}\ dx = \frac{q}{2\pi\varepsilon_0} ln\frac{D2}{D1} \qquad \textbf{III. 55}$$

Consider a single-phase line of 1 m length, made of two solid cylindrical conductors of radius r, separated by a distance D, as shown in Figure III.13

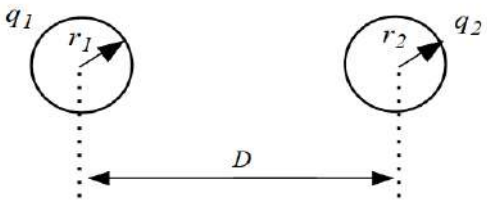


Figure III.13. Single-phase two-wire line.

Let us consider conductor 1 carrying a charge q_1 only. The voltage between conductor 1 and conductor 2 is:

$$V_{12(q1)} = \frac{q_1}{2\pi\varepsilon_0} ln\frac{D}{r} \qquad \textbf{III. 56}$$

Let us consider conductor 2 carrying a charge q_2 only. The voltage between conductor 2 and conductor 1 is:

$$V_{21(q2)} = \frac{q_2}{2\pi\varepsilon_0} ln\frac{D}{r} \qquad \textbf{III. 57}$$

Since $V_{12(q2)} = -V_{21(q2)}$ we obtain:

$$V_{12(q2)} = \frac{q_2}{2\pi\varepsilon_0} ln\frac{r}{D} \qquad \textbf{III. 58}$$

The potential difference due to the two charges is given by:

$$V_{12} = V_{12(q1)} + V_{12(q2)} = \frac{q_1}{2\pi\varepsilon_0} ln\frac{D}{r} + \frac{q_2}{2\pi\varepsilon_0} ln\frac{r}{D} \qquad \textbf{III. 59}$$

For a single-phase line: $q_2 = -q_1 = -q$ we obtain:

$$V_{12} = \frac{q}{\pi\varepsilon_0} ln\frac{D}{r} \qquad \textbf{III. 60}$$

So :

$$C_{12} = \frac{\pi\varepsilon_0}{ln\frac{D}{r}} \qquad \left(F/m\right) \qquad \textbf{III. 61}$$

C_{12} is the capacitance between conductors. To find the capacitance between each conductor and neutral.



(a) between two conductors

between conductors and point

Figure III.14. Capacity representation.

We know that $V_{1n} = \frac{1}{2} V_{12}$, Subsequently, $C = 2 \cdot C_{12}$ où :

$$C = \frac{2\pi\varepsilon_0}{ln\frac{D}{r}} \ (^F/_m) \qquad or \qquad C = \frac{0{,}0556}{ln\frac{D}{r}} \ (^{\mu F}/_{Km}) \qquad \textbf{III. 62}$$

The capacitance per phase is expressed using relations similar to those employed for the inductance per phase. However, unlike the calculation of inductance, where the Geometric Mean Radius (GMR) of the conductor is used, the capacitance formula directly involves the actual radius r of the conductor.

III.3.3.e Three-phase line capacity

Consider a three-phase line of unit length (1 m), composed of three cylindrical conductors of radius r, arranged according to the spacing shown in Figure III.15.

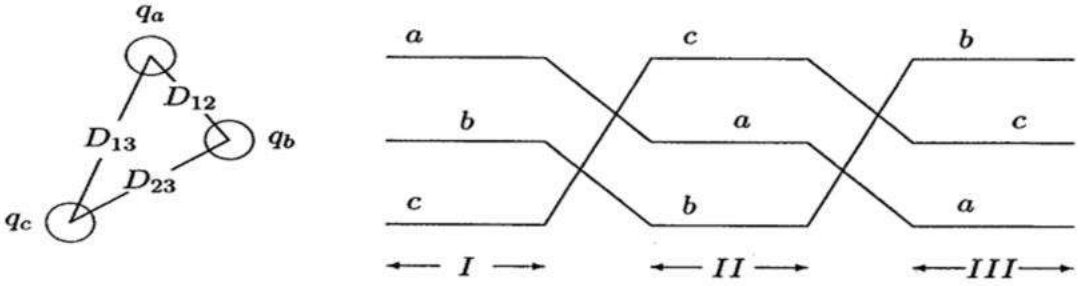


Figure III.15. Transposed three-phase line..

For a balanced system $q_a + q_b + q_c = 0$. Neglecting the effect of the ground and the shield wires, and assuming that the line is transposed:

$$V_{ab(I)} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{D_{12}}{\mathrm{r}} + q_b\, ln\frac{r}{D_{12}} + q_c\, ln\frac{D_{23}}{D_{13}}\right] \qquad \textbf{III. 63}$$

For the second section of the transposition:

$$V_{ab(II)} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{D_{23}}{\mathrm{r}} + q_b\, ln\frac{r}{D_{23}} + q_c\, ln\frac{D_{13}}{D_{12}}\right] \qquad \textbf{III. 64}$$

For the second section:

$$V_{ab(III)} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{D_{13}}{\mathrm{r}} + q_b\, ln\frac{r}{D_{13}} + q_c\, ln\frac{D_{12}}{D_{23}}\right] \qquad \textbf{III. 65}$$

The average value of $V_{ab}$is :

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{D_{12}D_{23}D_{13}}{r^3} + q_b\, ln\frac{r^3}{D_{12}D_{23}D_{13}} + q_c\, ln\frac{D_{12}D_{23}D_{13}}{D_{12}D_{23}D_{13}}\right] \qquad \textbf{III. 66}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{\mathrm{r}} + q_b\, ln\frac{r}{(D_{12}D_{23}D_{13})^{1/3}}\right] \qquad \textbf{III. 67}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{DMG}{\mathrm{r}} + q_b\, ln\frac{r}{DMG}\right] \quad \textbf{III. 68}$$

Where : $DMG = (D_{12}D_{23}D_{13})^{\frac{1}{3}}$

Similarly:

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left[q_a\, ln\frac{DMG}{\mathrm{r}} + q_c\, ln\frac{r}{DMG}\right] \quad \textbf{III. 69}$$

Substitute $-q_a = q_b + q_c$, we have:

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left[2q_a\, ln\frac{DMG}{\mathrm{r}} - q_a\, ln\frac{r}{DMG}\right] = \frac{3q_a}{2\pi\varepsilon_0} ln\frac{DMG}{\mathrm{r}} \quad \textbf{III. 70}$$

For balanced three-phase voltages:

$V_{ab} = V_{an}\angle 0° - V_{an}\angle -120°$, $V_{ac} = V_{an}\angle 0° - V_{an}\angle -240°$

So :

$V_{ab} + V_{ac} = 3V_{an}$

Subsequently, the capacity per phase to neutral:

$$C = {}^{q_a}/_{V_{an}} = \frac{2\pi\varepsilon_0}{ln\frac{DMG}{r}} \quad ({}^{F}/_{m}) \quad \textbf{III. 71}$$

$$C = \frac{0{,}0556}{ln\frac{DMG}{r}} \qquad ({}^{\mu F}/_{Km})$$

III.3.3.c Effect of bundled conductors

The procedure for determining the capacitance of a transposed three-phase line with bundled conductors follows the same steps as in the previous section:

$$C = \frac{2\pi\varepsilon_0}{ln\frac{DMG}{r^b}} \quad ({}^{F}/_{m}) \quad \textbf{III. 72}$$

$$C = \frac{0{,}0556}{ln\frac{DMG}{r^b}} \qquad ({}^{\mu F}/_{Km})$$

The effect of bundling is to introduce an equivalent radius r^b . $r^b$is similar to the GMR calculated for the inductance, except that the radius rrr of each sub-conductor is used instead of D_S. Thus:

For the bundle of two sub-conductors:

$$r^b = \sqrt[4]{(r\cdot d)^2} = \sqrt{r\cdot d} \quad \textbf{III. 73}$$

For the bundle of three sub-conductors:

$$r^b = \sqrt[9]{(r\cdot d\cdot d)^3} = \sqrt[3]{r\cdot d^2} \quad \textbf{III. 74}$$

For the bundle of four sub-conductors:

$$r^b = \sqrt[16]{\left(r \cdot d \cdot d \cdot d \cdot 2^{\frac{1}{2}}\right)^4} = 1{,}09\sqrt[4]{r \cdot d^3} \qquad \textbf{III. 75}$$

III.3.3.d Capacitance of a double-circuit three-phase line

Following the procedure of section (III.3.3.b), the average voltage V_a, V_b, V_c are calculated, and the equivalent capacitance per phase to neutral is obtained as:

$$C = \frac{2\pi\varepsilon_0}{ln\frac{DMG}{RMG_C}} \quad (^F/_m) \qquad \textbf{III. 76}$$

$$C = \frac{0{,}0556}{ln\frac{DMG}{RMG_C}} \qquad (^{\mu F}/_{Km})$$

The expression of the GMD is the same as that of the inductance between each group of phases (equation III.47).

$$D_{AB} = \sqrt[4]{D_{a1b1}.D_{a1b2}.D_{a2b1}.D_{a2b2}}$$
$$D_{BC} = \sqrt[4]{D_{b1c1}.D_{b1c2}.D_{b2c1}.D_{b2c2}} \qquad \textbf{III. 77}$$
$$D_{AC} = \sqrt[4]{D_{a1c1}.D_{a1c2}.D_{a2c1}.D_{a2c2}}$$

The equivalent GMD per phase is:$DMG = \sqrt[3]{D_{AB}D_{BC}D_{AC}}$

To find the equivalent GMRc between each group of phases, the procedure is the same as that used for the inductance GMRc, except that in equation (III.49) $r^b$is used instead of D_s^b .

This results in the following equations:

$$r_A = \sqrt{r^b D_{a1a2}}$$
$$r_B = \sqrt{r^b D_{b1b2}} \qquad \textbf{III. 78}$$
$$r_C = \sqrt{r^b{}_{c1c2}}$$

Where $r^b$is the geometric mean radius of the bundled conductors. The equivalent geometric mean radius for calculating the capacitance per phase to neutral is:

$$RMG_C = \sqrt[3]{r_A r_B r_C} \qquad \textbf{III. 79}$$

III.3.3.e Effect of the ground on capacitance

The ground acts as an equipotential surface, which leads to an increase in the line capacitance. The flux lines are necessarily perpendicular to it, and this effect can be accounted for using Kelvin's image charge method, particularly for the calculation of the capacitance of a three-phase line.

Let us consider in Figure III.16 a transposed line and conductor *a,b,c* carrying charges q_a, q_b, q_c , and occupying positions 1, 2, and 3, respectively, in the first part of the transposition cycle.

Below them are the conductors with the image charges $-q_a, -q_b, -q_c$, the capacitance to neutral is given by:

$$C = \frac{2\pi\varepsilon_0}{ln\frac{DMG}{r^b} - ln\frac{\sqrt[3]{H_{12}H_{23}H_{13}}}{\sqrt[3]{H_1H_2H_3}}} \quad \left(F/m\right) to\ neutral \qquad \textbf{III. 80}$$

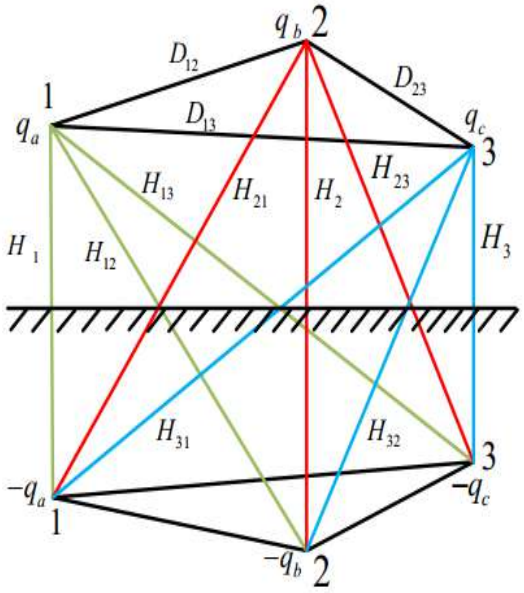


Figure III.16. Three-phase line and its image.

III.3.3.f Conductance due to the corona effect G

When a conductor is raised to a very high potential, the electric field in its vicinity can exceed the ionization threshold of the surrounding air. In this case, a luminous phenomenon in the form of a corona occurs around the conductor, particularly near its supports, known as the corona effect.

This effect mainly results from a leakage current between the conductor and the ground. It can be modeled by a per-unit-length cyclic conductance, denoted by G′ [S/km], and included in the equivalent circuit of the line. However, the value of this conductance generally remains low, particularly for voltages below 400 kV, which in most studies leads to it being neglected. The per-unit-length conductance of the overhead line is determined by:

$$g_0 = \frac{\Delta P}{U_n^2} \qquad Simmens/_{Km} \qquad \textbf{III.81}$$

ΔP: The per-unit-length power losses due to the corona effect.

U_n^2: The nominal voltage of the transmission line.

The total conductance of the overhead line is equal to:

$$G = g_0 1\ (Simmens)\ ou\ \Omega^{-1} \qquad \textbf{III .82}$$

III.4. Circuit Equivalent of Transmission Lines

III.4.1. Modeling and Performance of Transmission Lines

Transmission lines are represented by an equivalent model with the appropriate **per-phase** circuit parameters. The model used to calculate voltages, currents, and power flows depends on the length of the line. Depending on their length, electrical transmission lines are generally classified, in most countries, as follows:

- **Short line**: length less than 80 km (l < 80 km).
- **Medium line:** length between 80 km and 250 km (80 ≤ l <250 Km).
- **Long line:** length greater than or equal to 250 km (l ≥ 250 km).

A transmission line can be modeled as a **two-port network** (Figure III.17). This model allows establishing a relationship between the electrical quantities at the two ends of the line, in which the sending-end voltage and current (V_S and I_S) are related to the receiving-end voltage and current (V_R and I_R) by the following matrix equation:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad \textbf{III.83}$$

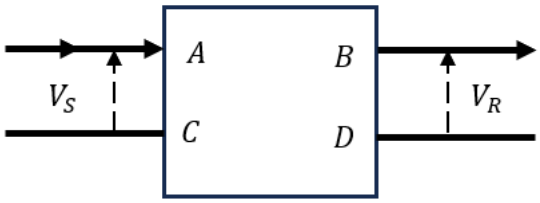


Figure III.17. Representation of a transmission line as a two-port network.

The determinant of the matrix system $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is equal to 1 :

$$AD - BC = 1 \qquad \textbf{III.84}$$

In transmission line modeling, the per-unit-length parameters are denoted in lowercase, while the total parameters (corresponding to the entire line length) are denoted in uppercase.

$z = r + jL\omega\ (\Omega/Km)$ Impédance série par unité de longueur

$y = g + jC\omega\ (S/Km)$ Admittance de shunt par unité de longueur

$Z = z \cdot l\ (\Omega)$ Impédance totale de la ligne

$Y = y \cdot l\ (S)$ Admittance totale de la ligne

l = longueur de ligne (km).

III.4.1.A A Model of a Short Transmission Line

For a short line (length < 80 km or voltage < 69 kV), the distributed capacitance can be neglected. The equivalent model is then reduced to a series impedance (Z=R+jωL) representing the total resistance and inductance of the line.

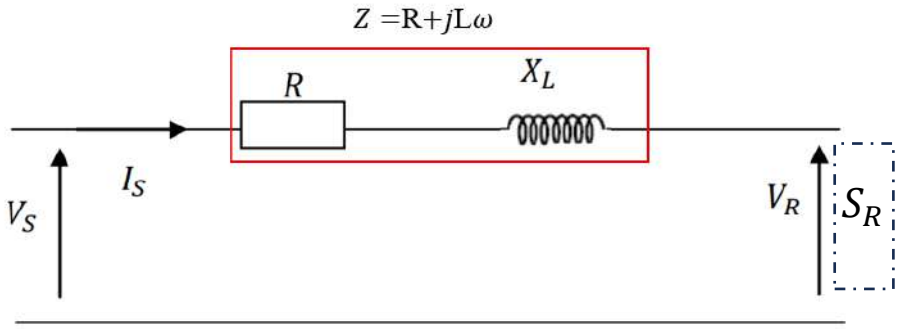


Figure III.18. Equivalent circuit of a short transmission line.

If a three-phase load with apparent power S_R is connected at the receiving end of the transmission line, the current at the receiving end is:

$$I_R = \frac{S^*_{(3\phi)}}{3V^*_R} \quad \textbf{III.85}$$

The sending-end voltage is:

$$V_S = V_R + ZI_R \quad \textbf{III.86}$$

The shunt impedance is neglected, so:

$$I_S = I_R \quad \textbf{III.87}$$

By applying Kirchhoff's laws, we obtain:

$$\begin{cases} V_S = V_R + (R + jX_L) \cdot I_R \\ I_S = I_R \end{cases} \quad \textbf{III.88}$$

$$\begin{cases} V_S = V_R + Z \cdot I_R \\ I_S = I_R \end{cases} \quad \textbf{III.89}$$

Or in matrix form:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \textbf{III.90}$$

By comparison with the matrix form, we find:

$A = D = 1$, $B = Z$, $C = 0$

Once the sending-end voltage is calculated, the sending-end power is obtained by:

$$S_{S(3\phi)} = 3V_S I^*_S \quad \textbf{III.91}$$

The total line losses are given by:

$$S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)} \quad \textbf{III. 92}$$

The efficiency of the transmission line is given by:

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \qquad \textbf{III. 93}$$

$P_{S(3\phi)}$, $P_{R(3\phi)}$represent, respectively, the total active power at the sending end (source side) and at the receiving end (load side) of the transmission line.

III.4.1.B Model of a Medium Transmission Line

For a medium-length transmission line (80–250 km), the capacitive effect can no longer be neglected. The shunt admittance is then taken into account and divided into two equal parts placed at the ends, which defines the nominal π-model, based on the series impedance Z and the shunt admittance Y.

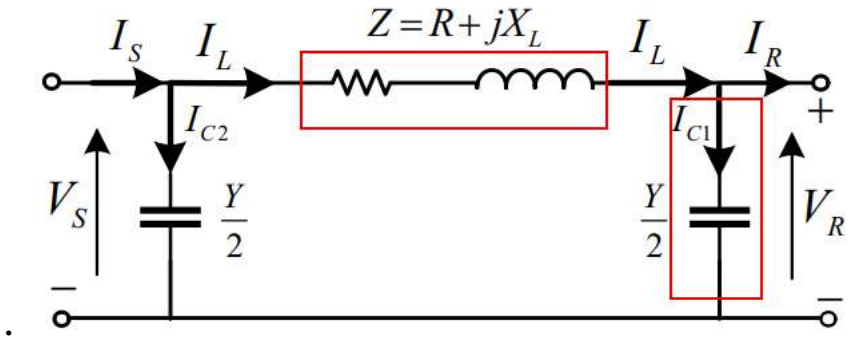


Figure III.19. Nominal π-model of a medium-length transmission line.

By applying Kirchhoff's current and voltage laws:

$$\begin{cases} I_L = \mathrm{I_R} + \frac{Y}{2}\mathrm{V_R} \\ V_S = \mathrm{V_R} + ZI_L \end{cases} \qquad \mathrm{V_S} = \left(1 + \tfrac{\mathrm{ZY}}{2}\right)\mathrm{V_R} + Z\mathrm{I_R} \qquad \textbf{III.94}$$

The sending-end current is:

$$I_S = \mathrm{I_L} + \frac{Y}{2}\mathrm{V_S} \qquad \textbf{III. 95}$$

By substituting for $\mathrm{I_L}$ and $\mathrm{V_S}$:

$$I_S = Y\left(1 + \frac{\mathrm{ZY}}{4}\right) + \left(1 + \frac{\mathrm{ZY}}{2}\right)\mathrm{I_R} \qquad \textbf{III. 96}$$

For an ABCD two-port representation of the nominal π-model, we have:

$$A = 1 + \frac{\mathrm{ZY}}{2}; \qquad B = \mathrm{Z}; \qquad \textbf{III. 97}$$

$$C = Y\left(1 + \frac{\mathrm{ZY}}{4}\right); \qquad D = 1 + \frac{\mathrm{ZY}}{2};$$

III.4.1.C Model of a Long Transmission Line

For short and medium transmission lines (up to 250 km), reasonably accurate models are obtained by assuming that the line parameters are concentrated at the ends. Thus, for a medium-length line (80–250 km), the capacitive effect is taken into account by splitting the shunt admittance into two parts at the ends, leading to the nominal π-model, defined by the series impedance Z and the shunt admittance Y.

For long lines (≥ 250 km), a more precise approach is required: the exact effect of the distributed parameters along the line must be considered. In this case, the expressions for voltages and currents at every point along the line are derived, from which an equivalent π-model can be obtained to represent the long line. Figure III.20 shows one phase of a distributed line of length l km.

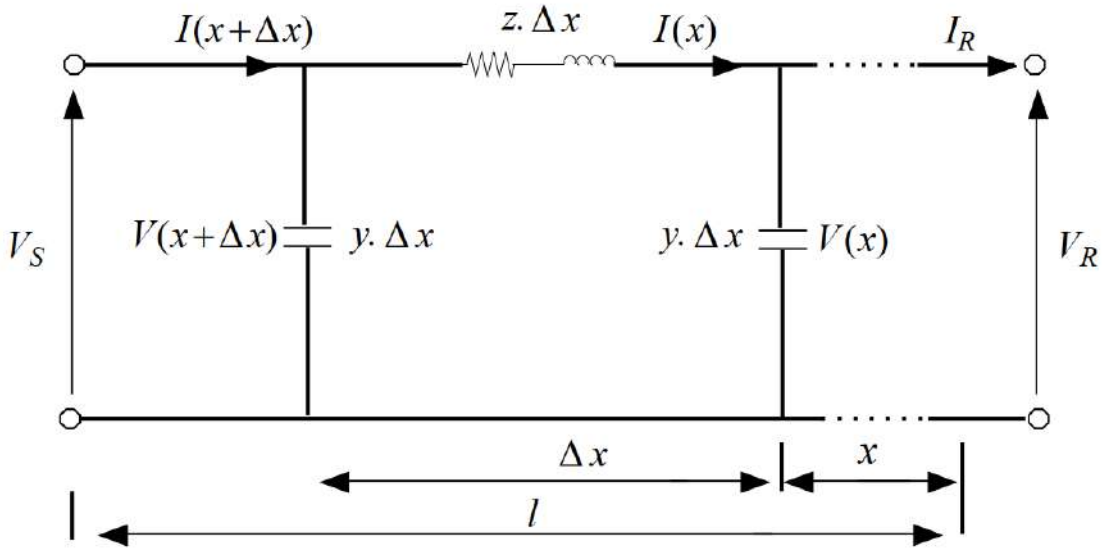


Figure III.20. Long transmission line with distributed parameters.

The total series impedance and shunt admittance of the line are:

$$\begin{cases} Z = (r + j\omega L)l \\ Y = (g + j\omega C)l \end{cases} \quad \textbf{III. 98}$$

✓ **For the voltages**

$$V(x + \Delta x) = V(x) + z\Delta x \cdot I(x) \quad \textbf{III. 99}$$

$$\frac{V(x + \Delta x) - V(x)}{\Delta x} = z \cdot I(x) \quad \textbf{III. 100}$$

Comme

$$\Delta x \to 0: \frac{dV(x)}{dx} = z \cdot I(x) \quad \textbf{III. 101}$$

✓ **For the currents**

$$I(x+\Delta x) = \mathrm{I}(x) + y\Delta x \cdot V(x+\Delta x) \qquad \textbf{III.102}$$

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y \cdot V(x+\Delta x) \qquad \textbf{III.103}$$

as:

$$\Delta x \to 0: \frac{dI(x)}{dx} = y \cdot V(x) \qquad \textbf{III.104}$$

By differentiating equation (III.101) and substituting from (III.104):

$$\frac{d^2V(x)}{dx^2} = z\frac{dI(x)}{dx} = zy \cdot V(x) \qquad \textbf{III.105}$$

Let us define:

$$\gamma^2 = zy \qquad \textbf{III.106}$$

$$\gamma = \sqrt{zy}$$

$$\frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0 \qquad \textbf{III.107}$$

The solution of this equation is of the form:

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \qquad \textbf{III.108}$$

Where γ is the propagation constant

Where:

$$\gamma = \alpha + j\beta = \sqrt{zy} = \sqrt{(r + j\omega L)(g + j\omega C)} \qquad \textbf{III.109}$$

The real part α is the attenuation constant, and the imaginary part β is the phase constant measured in radians per unit length.

From equation (III.101), the current is:

$$I(x) = \frac{I}{z}\frac{dV(x)}{dx} = \frac{\gamma}{z}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) = \sqrt{\frac{y}{z}}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \qquad \textbf{III.110}$$

Where

$$I(x) = \frac{1}{Z_C}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \qquad \textbf{III.111}$$

Where Z_C is the characteristic impedance, given by:

$$Z_C = \sqrt{\frac{z}{y}} \qquad \textbf{III.112}$$

To find A_1 et A_2 , we note that when :$x = 0$, $V(x) = V_R$, $I(x) = I_R$

Starting from (III.108) and (III.111), these constants turn out to be:

$$A_1 = \frac{V_R + Z_C I_R}{2} \qquad \textbf{III.113}$$

$$A_2 = \frac{V_R - Z_C I_R}{2}$$

Substituting into (III.108) and (III.111), the general expression for voltage and current along a long transmission line becomes:

$$V(x) = \frac{V_R + Z_C I_R}{2} e^{\gamma x} + \frac{V_R - Z_C I_R}{2} e^{-\gamma x} \quad \textbf{III.114}$$

$$I(x) = \frac{\frac{V_R}{Z_C} + I_R}{2} e^{\gamma x} + \frac{\frac{V_R}{Z_C} - I_R}{2} e^{-\gamma x} \quad \textbf{III.115}$$

The equations for voltage and current can be rearranged as follows:

$$V(x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_C \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R \quad \textbf{III.116}$$

$$I(x) = \frac{1}{Z_C} \frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R \quad \textbf{III.117}$$

Recognizing the hyperbolic functions sinh and cosh, the above equation can be written as follows:

$$V(x) = \cosh \gamma x \, V_R + Z_C \sinh \gamma x \, I_R \quad \textbf{III.118}$$

$$I(x) = \frac{1}{Z_C} \sinh \gamma x \, V_R + \cosh \gamma x \, I_R \quad \textbf{III.119}$$

For a quadripole ABCD notation for the nominal π model, we have:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Where $A = \cosh \gamma x$; $B = Z_C \sinh \gamma x$; $C = \frac{1}{Z_C} \sinh \gamma x$; $D = \cosh \gamma x$;

It is now possible to find an exact equivalent π model to replace the ABCD constants of the quadrupole. Similar to the expressions for a mean line obtained for the nominal π model, for the equivalent π model we have:

$$\mathrm{V_S} = \left(1 + \frac{Z' Y'}{2}\right) \mathrm{V_R} + Z \mathrm{I_R} \quad \textbf{III.120}$$

$$I_S = Y \left(1 + \frac{Z'Y'}{4}\right) + \left(1 + \frac{ZY}{2}\right) \mathrm{I_R}$$

Table III.1. Summary table

Types	Equivalent circuit	A	B	C	D
Short	Series impedance	1	Z	0	A

Moyenne	Circuit in π	$1+\frac{ZY}{2}$	Z	$Y(1+\frac{ZY}{4})$	$1+\frac{ZY}{2}$
	Circuit in T	$1+\frac{ZY}{2}$	$Z(1+\frac{ZY}{4})$	Y	$1+\frac{ZY}{2}$
Long	Distributed constant circuit	$cosh\,\gamma x$	$Z_C\,sinh\,\gamma x$	$\frac{1}{Z_C}sinh\,\gamma x$	$cosh\,\gamma x$

Voltage regulation:

Line voltage regulation is the increase in voltage at the receiving end expressed as a percentage relative to the full load voltage, with the source voltage kept constant.

$$\boldsymbol{Reg}\ (\%) = \frac{V_{R(0)} - V_{R\,(PC)}}{V_{R\,(PC)}} \qquad \textbf{III.121}$$

Where :

$V_{R(O)}$ is the voltage value at no-load reception.

$V_{R\,(PC)}$: Full load voltage value.

Table III.2. Voltage regulation

Lignes	Regulation %
Short	$100 \cdot (V_S - V_R)/V_R$
Moyenne	$100 \cdot (V_S/A - V_R)/V_R$
Long	$100 \cdot (V_S/A - V_R)/V_R$

III.5. Calculation of voltage drops

The voltage drop in an electrical network is caused by ohmic and reactive losses in the lines and depends on the load current, resistance R, reactance, and power factor$(\cos\varphi)$. Excessive voltage drop reduces the voltage at the load terminals and can affect system performance and stability. To ensure good power quality, it should generally remain below 3–5 %.

III.5.1. Calculation of voltage drop ΔU

✓ **Two-phase:**

$$\Delta U = 2I_B L(R\cos\varphi + X\sin\varphi) \qquad \textbf{III.122}$$

In % : $100\,{}^{\Delta U}/_{U_n}$

✓ **Single-phase phase and neutral**

$$\Delta U = 2I_B L(R\cos\varphi + X\sin\varphi) \qquad \textbf{III. 123}$$

In % : $100\,{}^{\Delta U}/_{V_n}$

✓ **Three-phase (with or without neutral)**

$$\Delta U = \sqrt{3}I_B L(R\cos\varphi + X\sin\varphi) \qquad \textbf{III. 124}$$

In % : $100\,{}^{\Delta U}/_{U_n}$

Where :

I_B : operating current in amperes;

U_n : nominal voltage between phases;

V_n: nominal voltage between phase and neutral;

L : cable length in kilometers;

R : linear resistance of a conductor in Ω/km ;

X : linear reactance of a conductor in Ω/km ;

X is negligible for cables of cross-section $S < 50$ mm^2

In the absence of other indications, the following will be taken $X = 0.08\Omega / km$.

$R = \frac{22{,}5\Omega.\text{mm}^2/\text{Km}}{S(\text{section en mm}^2)}$ For copper;

$R = \frac{36\Omega.\text{mm}^2/\text{Km}}{S(\text{section en mm}^2)}$ For aluminum;

Note: R is negligible beyond a section of 500 mm^2

φ : phase shift of the current relative to the voltage in the circuit under consideration; generally:

- Lighting : cos φ =1 ;
- Electromotive force (EMF):
  - During startup: cos φ =0.35 ;
  - In normal service: cos φ =0.8 ;

III.5.2 Calculation from a simplified table

Voltage drop tables are used to calculate the voltage drop along a conductor. The considered parameters include:

• Conductor material (copper or aluminum)

• Cross-section (mm²)

• Current carried (A)

• Load power factor

- ☞ The values are provided for single-phase and three-phase networks.
- ☞ They are expressed in V/km/A, which facilitates the sizing and verification of installations.

The voltage drop can then be expressed as:

$\Delta U = K \cdot I_B \cdot L \ \ en\ (V)$

- K : given by the table
- I_B : Operating current in amperes,
- L : Cable length in km.

Table III.3. Voltage drop ΔU in volts / ampere / km in a circuit

Section in mm²		Single-phase circuit			Balanced three-phase circuit		
		Electromotive force		Lighting	Electromotive force		Lighting
Cop	Al	Normal service cos φ = 0,8	Startup cos φ = 0,35	cos φ = 1	Normal service cos φ = 0,8	Startup cos φ = 0,35	cos φ = 1
1,5		24	10,6	30	20	9,4	25
2,5		14,4	6,4	18	12	5,7	15
4		9,1	4,1	11,2	8	3,6	9,5
6	10	6,1	2,9	7,5	5,3	2,5	6,2
10	16	3,7	1,7	4,5	3,2	1,5	3,6
16	25	2,36	1,15	2,8	2,05	1	2,4
25	35	1,5	0,75	1,8	1,3	0,65	1,5
35	50	1,15	0,6	1,29	1	0,52	1,1
50	70	0,86	0,47	0,95	0,75	0,41	0,77
70	120	0,64	0,37	0,64	0,56	0,32	0,55
95	150	0,48	0,30	0,47	0,42	0,26	0,4
120	185	0,39	0,26	0,37	0,34	0,23	0,31
150	240	0,33	0,24	0,30	0,29	0,21	0,27
185	300	0,29	0,22	0,24	0,25	0,19	0,2
240	400	0,24	0,2	0,19	0,21	0,17	0,16
300	500	0,21	0,19	0,15	0,18	0,16	0,13

III.6. Ferranti Effect

The Ferranti effect occurs when a long transmission line is lightly loaded or left open-circuited at the receiving end. In this situation, the voltage at the receiving end can become higher than the sending-end voltage. This phenomenon is due to the line's charging current: under alternating voltage, the distributed capacitance of the line generates a charging current, also called capacitive current. The longer the line and the higher the voltage, the more significant this effect becomes.

Neglecting losses, this overvoltage is given by:

$$\frac{U_R}{U_S} = \frac{1}{\cos(\sqrt{LC}\omega)} \qquad \textbf{III. 125}$$

For short line lengths (a few hundred km at 50 Hz), it can also be approximated by:

$$\frac{U_R}{U_S} = \frac{1}{1 - \frac{LC\omega^2}{2}} \qquad \textbf{III. 126}$$

III.6.1. Mitigation of the Ferranti Effect

The Ferranti effect, which causes overvoltage at the receiving end of long lines under light load or no load, can be mitigated by several methods:

- **Insertion of shunt inductive reactances (shunt reactors) at the receiving end:**
  → They compensate for the line's capacitive current and reduce the overvoltage.
- **Use of transformers with on-load tap changers (OLTC):**
  → They allow adjusting the sending-end voltage to keep the receiving-end voltage within acceptable limits.
- **FACTS compensation (e.g., SVC, STATCOM):**
  → They provide dynamic reactive power regulation, thus enabling effective voltage control even under transient conditions.
- **Optimization of line operation:**
- → Avoid leaving long lines completely unloaded, or take them out of service when they are not needed.

III.7. Transmitted Power and Power Factor Compensation in Lines

III.7.1. Power Transmitted by a Line

The specific expressions for complex power flow on a line can be obtained in terms of the sending and receiving voltages, phase angles, and ABCD constants.
The ABCD constants in polar form are given as:

$$A = |A|\angle\theta_A, \qquad B = |B|\angle\theta_B,$$

The source voltage: $V_S = |V_S|\angle\delta$ and the load voltage as a reference $V_R = |V_R|\angle 0$.

From equation (III.90), the current I_R can be written as follows:

$$I_R = \frac{|V_S|\angle\delta - |A|\angle\theta_A\,|V_R|\angle 0}{|B|\angle\theta_B} = \frac{|V_S|}{|B|}\angle\delta - \theta_B - \frac{|A||V_R|}{|B|}\angle\theta_A - \theta_B \qquad \textbf{III. 127}$$

The complex power at the receiving end:

$$S_{R(3\phi)} = P_{R(3\phi)} + jQ_{R(3\phi)} 3V_R I_R^* \qquad \textbf{III. 128}$$

By substituting I_R from (III.126), we get:

$$S_{R(3\phi)} = 3\frac{|V_S||V_R|}{|B|}\angle\theta_B - \delta - 3\frac{|A||V_R|^2}{|B|}\angle\theta_B - \theta_A \quad \textbf{III.129}$$

In terms of line-to-line (phase-to-phase) voltages, we have:

$$S_{R(3\phi)} = \frac{\left|V_{S(L-L)}\right|\left|V_{R(L-L)}\right|}{|B|}\angle\theta_B - \delta - \frac{|A|\left|V_{R(L-L)}\right|^2}{|B|}\angle\theta_B - \theta_A \quad \textbf{III.130}$$

The active and reactive powers at the receiving end of the line are:

$$P_{R(3\phi)} = \frac{\left|V_{S(L-L)}\right|\left|V_{R(L-L)}\right|}{|B|}\cos(\theta_B - \delta) - \frac{|A|\left|V_{R(L-L)}\right|^2}{|B|}\cos(\theta_B - \theta_A) \quad \textbf{III.131}$$

$$Q_{R(3\phi)} = \frac{\left|V_{S(L-L)}\right|\left|V_{R(L-L)}\right|}{|B|}\sin(\theta_B - \delta) - \frac{|A|\left|V_{R(L-L)}\right|^2}{|B|}\sin(\theta_B - \theta_A) \quad \textbf{III.132}$$

The complex power of the source is:

$$S_{S(3\phi)} = P_{S(3\phi)} + jQ_{S(3\phi)} 3V_S I_S^* \quad \textbf{III.133}$$

I_S can be written as:

$$I_R = \frac{|A|\angle\theta_A\,|V_S|\angle\delta - |V_R|\angle 0}{|B|\angle\theta_B} \quad \textbf{III.134}$$

By substituting I_S into (III.132), we obtain:

$$P_{S(3\phi)} = \frac{|A|\left|V_{S(L-L)}\right|^2}{|B|}\cos(\theta_B - \theta_A) - \frac{\left|V_{S(L-L)}\right|\left|V_{R(L-L)}\right|}{|B|}\cos(\theta_B + \delta) \quad \textbf{III.135}$$

$$Q_{S(3\phi)} = \frac{|A|\left|V_{S(L-L)}\right|^2}{|B|}\sin(\theta_B - \theta_A) - \frac{\left|V_{S(L-L)}\right|\left|V_{R(L-L)}\right|}{|B|}\sin(\theta_B + \delta) \quad \textbf{III.136}$$

The active and reactive losses of the transmission line:

$$P_{L(3\phi)} = P_{S(3\phi)} - P_{R(3\phi)} \quad \textbf{III.137}$$

$$Q_{L(3\phi)} = Q_{S(3\phi)} - Q_{R(3\phi)} \quad \textbf{III.138}$$

III.8. Power Factor Compensation

III.8.1.A Reactive Power Compensation

Reactive power compensation is an essential technique in electrical systems aimed at improving power quality and transmission efficiency. It involves installing devices capable of supplying or absorbing the reactive power required by the network, in order to maintain the power factor close to unity and to limit losses.

The main objectives of reactive power compensation are:

- **Improvement of the power factor:** reduction of the reactive current flowing in the lines.
- **Reduction of Joule losses**: less total current flowing means lower losses in the conductors.
- **Voltage stabilization:** maintaining an adequate voltage profile in the network, especially in case of large load variations.
- **Increase of line transfer capacity:** freeing up capacity by reducing reactive current allows more active power to be transmitted**.**

The means of compensation can be:

- **Passive:** capacitor banks, reactance coils (shunt or series reactors).
- **Active:** FACTS devices such as SVC (Static Var Compensator) and STATCOM (Static Synchronous Compensator), providing fast and dynamic reactive power regulation.

III.8.1.B Overview

Any alternating current electrical machine (motor, transformer, etc.) involves two main forms of energy:

✓ **Active energy: c**orresponds to the active power P (expressed in kW). It is converted into useful mechanical energy (work) and heat due to losses.

$$P = UI\cos(\varphi) \quad (Kw) \qquad \textbf{III. 139}$$

✓ **Reactive energy:** corresponds to reactive power Q (expressed in kVAR). It is necessary for establishing magnetic fields in electrical devices (transformers, rotating machines, etc.).

$$Q = UI\sin(\varphi) \quad (KVar) \qquad \textbf{III. 140}$$

✓ **Apparent energy:** the distribution network supplies the apparent power S (expressed in kVA), which results from the combination of active and reactive powers according to the following relation:

$$S = UI = \sqrt{P^2 + Q^2} \quad (KVA) \qquad \textbf{III. 141}$$

The three powers P, S, and Q are represented vectorially by a power triangle:

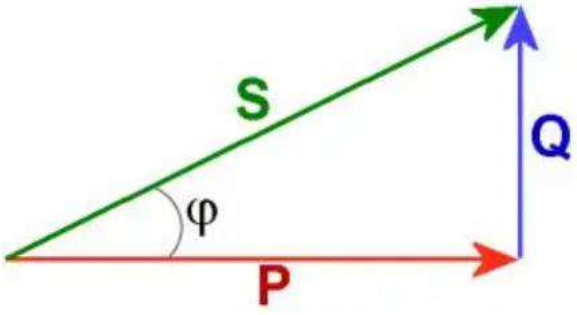


Figure III.21. Power triangle.

III.8.1.C Power Factor

The power factor (PF), usually denoted as cos φ, expresses the ratio between the active power actually used and the apparent power supplied by the network:

$$\cos\varphi = \frac{P}{S} = \frac{Active\ power\ (kW)}{Apparent\ power(KVA)} \qquad \textbf{III. 142}$$

The cos(φ) ranges between 0 and 1.

A power factor close to 1 improves the efficiency of the installation by reducing unnecessary reactive power and optimizing the use of the network.

The tangent of the phase angle can be written as φ between voltage and current in terms of power:

$$\tan\varphi = \frac{Q}{P} \qquad \textbf{III. 143}$$

III.8.2. Advantage of Power Factor Improvement

- **Reduction of the electricity bill**

For large consumers, reactive energy is considered in billing when it exceeds a certain threshold, particularly during defined periods. Energy distributors apply specific tariff schedules that vary according to consumption. Reducing reactive energy consumption, achieved by improving the power factor, therefore has a direct and significant impact on the electricity bill.

In order to encourage subscribers to locally compensate part of their reactive power, electricity providers apply a bonus-malus system based on the power factor; for SONELGAZ:

- ☞ $\cos\varphi \geq 0{,}894$ The customer will receive a bonus.
- ☞ $\cos\varphi < 0{,}894$ The customer will be penalized.

- **Optimization of technical and economic choices**

Improving the power factor offers several technical and economic advantages for electrical installations:

- **Reduction in cable cross-section:** a low power factor requires cables with a larger cross-section to carry the same active power.
- **Reduction of line losses:** a good power factor limits the current flowing, which reduces Joule losses, which are proportional to the square of the current.
- **Reduction in voltage drop:** reducing the reactive power carried helps to reduce voltage drops in the lines**.**
- **Increased available power:** a high power factor increases the usable active power at the secondary side of transformers.

III.8.3.Principle of compensation

Compensating an installation involves installing a source of reactive power that helps improve the power factor. This compensation is achieved by locally supplying the required reactive power. The objective of reactive power compensation is to reduce the current drawn from the network. Reactive power is supplied by capacitors, as close as possible to the inductive loads. Each of the active and reactive powers corresponds to a current.

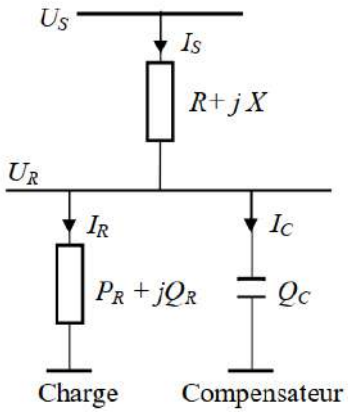


Figure III.22. Local compensation at the load level.

The figure below shows the vector representation of compensation

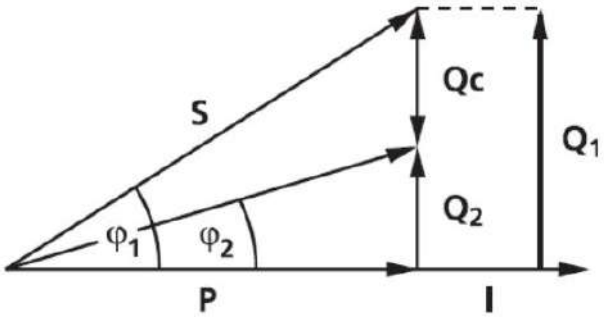


Figure III.23. Vector representation of active and reactive power before and after compensation.

✓ **Before compensation:**

$$S_1 = \sqrt{\mathrm{P}^2 + Q_1^2} \quad \rightarrow \quad \tan\varphi_1 = \frac{Q_1}{P} \qquad \textbf{III. 143}$$

✓ **After compensation :**

$$S_2 = \sqrt{\mathrm{P}^2 + Q_2^2} \quad \rightarrow \quad \tan\varphi_2 = \frac{Q_2}{P} \qquad \textbf{III. 145}$$

The reactive power compensation for constant active power is:

$$Q_C = Q_2 - Q_1 = P \cdot \tan\varphi_2 - P \cdot \tan\varphi_1 \qquad \textbf{III. 146}$$

The general expression is:

$$Q_C = P \cdot (\tan\varphi_2 - \tan\varphi_1) \qquad \textbf{III. 147}$$

III.8.4. Compensation methods

Reactive power compensation can be carried out in various ways, depending on the type of installation, the load profile, and the desired objectives::

✓ Global compensation: The capacitor bank is connected at the head of the installation and remains in service permanently. This mode is suitable when the load is stable and continuous.

- ✓ Partial compensation: The capacitor bank is connected to the distribution panel and supplies reactive power per workshop or per group of loads. This mode is suitable for large installations with multiple workshops having different load profiles.
- ✓ Individual compensation: The capacitor bank is connected directly to the terminals of each inductive load, particularly motors. This mode is recommended when certain loads have significant power compared to the total power, as it provides the best benefits in terms of efficiency and loss reduction.

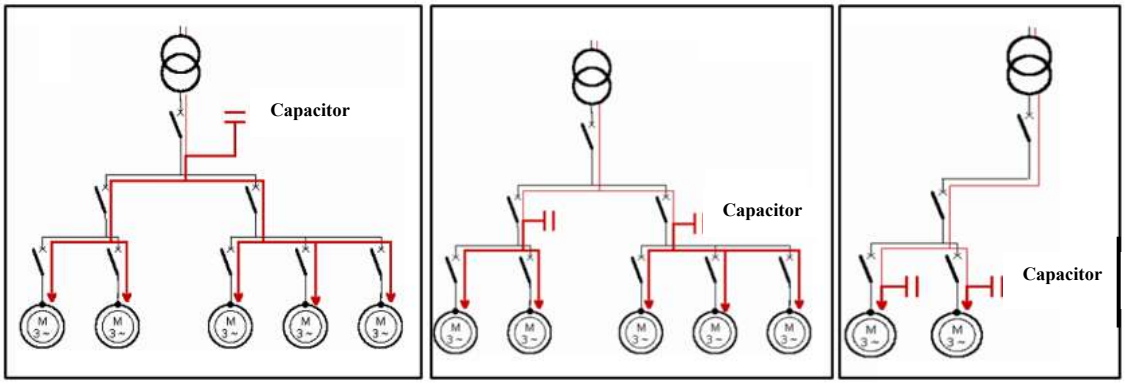


Figure III.24. Various options for installing capacitor batteries.

III.8.5. Calculation of capacitor values according to the coupling mode

A. Triangle coupling of capacitors

The reactive power supplied by a capacitor:

$$|Q_C| = C\omega U^2 \qquad \textbf{III. 148}$$

Reactive power absorbed by the three capacitors:

$$|Q_{C\,tot}| = 3|Q_C| = 3C\omega U^2 \qquad \textbf{III. 149}$$

The capacitor capacity is calculated as follows:

$$C_\Delta = \frac{P\,(\tan\varphi_2 - \tan\varphi_1)}{3\omega U^2} \qquad \textbf{III. 150}$$

B. Star connection of capacitors

Using the same reasoning as before,

$$|Q_{C\,tot}| = 3|Q_C| = 3C\omega V^2 \qquad \textbf{III. 151}$$

It is shown that the capacitance of the capacitor is given by the relationship:

$$C_{étoile} = \frac{P\,(\tan\varphi_2 - \tan\varphi_1)}{\omega U^2} \qquad \textbf{III. 152}$$

Star connection is less advantageous because it requires a capacitor capacity three times higher than that needed for a delta connection (Figure III.25). A larger capacity means bigger capacitors and therefore higher costs.

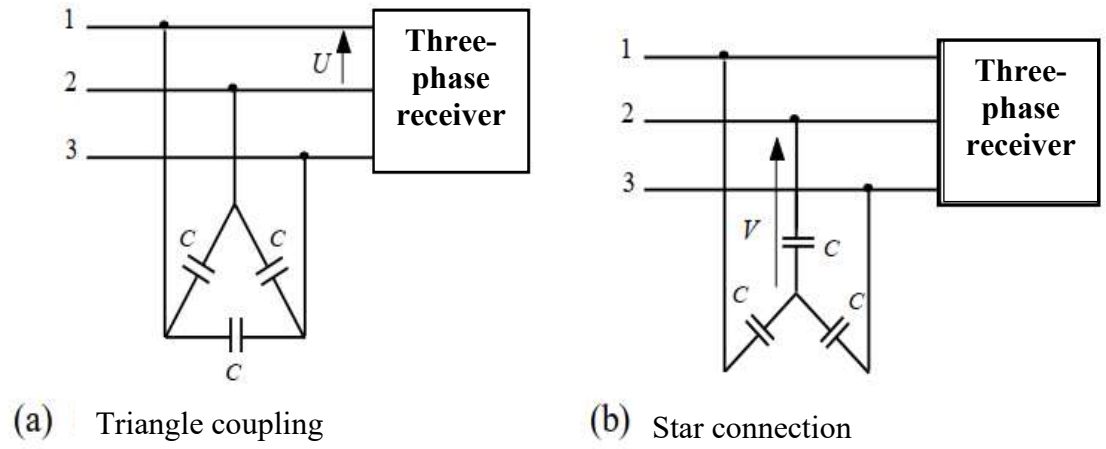


Figure III.25. Compensation battery coupling modes.

Chapter IV

Transformers and relative unit system

IV.1. Fundamentals of the Transformer

Transformers are entirely static machines that, in alternating current, allow the modification of electrical quantities — voltage and current — without changing either the frequency or the waveform.The absence of moving parts contributes to their high efficiency. Several types of transformers can be distinguished, notably single-phase and three-phase. These devices play a central role in electrotechnics, as they form the basic equipment for the transmission and distribution of electrical energy.

They operate at two main levels:

- Voltage step-up: between the generation source and the transmission network, in order to limit losses during long-distance transmission.
- Voltage step-down: from the network to end-users, in order to supply a voltage suitable for final consumption.

IV.2. Single-Phase Transformer

A single-phase transformer is a static electrical machine designed to transfer electrical energy from one circuit to another in alternating current, by modifying the voltage and current while keeping the same frequency. It mainly consists of two windings:

- Primary winding: connected to the AC supply source of voltage V_1 and consisting of n_1 turns. It draws the current I_1.
- Secondary winding: connected to the load, it consists of n_2 turns; under the voltage V_2, it supplies a current I_2 to the receiving device.

IV.2.1. Principle of Operation

The principle of operation is based on electromagnetic induction : a variation of the magnetic flux in the transformer core induces a voltage in the secondary winding.

One of the two windings acts as the primary; it is supplied by a variable voltage and generates a variable flux in the magnetic circuit. The magnetic circuit conducts, with the lowest possible

reluctance, the magnetic field lines created by the primary through the turns of the second winding.

According to Faraday's law, this varying magnetic flux induces an electromotive force (EMF) in the second winding, which is called the secondary of the transformer.

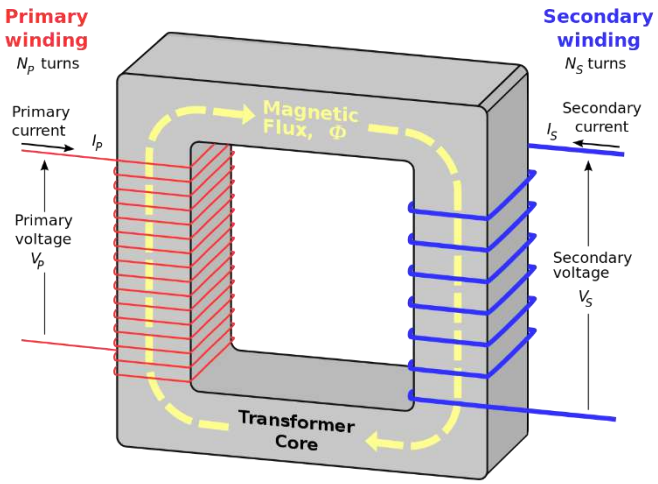


Figure IV.1. Equivalent circuit of the single-phase transformer.

By its principle, the transformer cannot operate if it is supplied with a direct voltage. The flux must be variable to induce an EMF in the secondary; therefore, the primary voltage must be alternating.

IV.2.2. Symbol

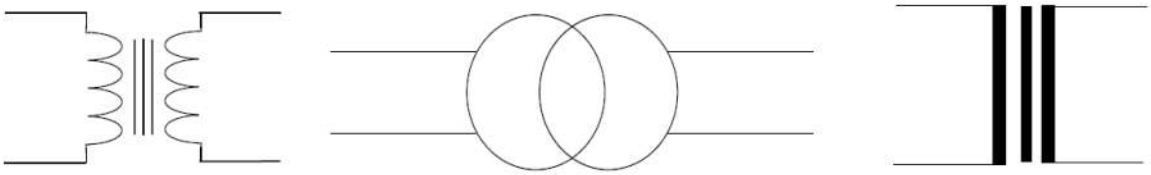


Figure IV.2. Single-phase transformer symbols.

IV.3. Perfect Transformer

Perfect: there are no losses; the efficiency is 100%;

IV.3.1. Simplifying Assumptions

A transformer is considered perfect if we neglect**:**

- The winding resistances (no Joule losses);

- ❖ The leakage inductances ;
- ❖ The reluctance of the magnetic circuit;
- ❖ No iron losses.

IV.3.2. Relationships between voltages

At every instant, each turn of the transformer is crossed by the same magnetic flux, which implies the absence of magnetic leakage.

The flux through a section of the magnetic circuit is denoted φ(t).

✓ The total flux φ1 for the primary winding. $\varphi_1(t) = N_1\varphi(t)$

✓ The total flux φ2 for the secondary winding. $\varphi_2(t) = N_2\varphi(t)$

A. At the primary

Consisting of N1 turns, it is the seat of an EMF:

$$e_1 = -N_1 \frac{d\phi}{dt} \qquad \textbf{IV.1}$$

$$u_1 = -e_1$$

In complex form:

$$\underline{U_1} = -\underline{E_1} = j\omega N_1 \underline{\phi} \qquad \textbf{IV.2}$$

B. At the secondary

Consisting of N2 turns, it is the seat of an EMF:

$$e_2 = -N_2 \frac{d\phi}{dt} \qquad \textbf{IV. 3}$$

$$u_2 = e_2$$

In complex form:

$$\underline{U_2} = \underline{E_2} = j\omega N_2 \underline{\phi} \qquad \textbf{IV.4}$$

It follows immediately:

$$\frac{u_2}{u_1} = -\frac{e_2}{e_1} = -\frac{u_2}{u_1} = -m \qquad \textbf{IV. 5}$$

The voltages u_1 and u_2 are in phase opposition.

The relationship between the RMS values U1 and U2 does not take the phase shift into account:

$$\frac{U_2}{U_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = m \qquad \textbf{IV.6}$$

m: transformation ratio of the transformer.

Depending on the value of mmm, we can distinguish:

- If m > 1($U_2 > U_1$): the transformer is a step-up transformer;
- If m < 1 ($U_2 < U_1$): the transformer is a step-down transformer.
- If m =1 ($U_2 = U_1$): The transformer provides electrical isolation between the source and the load.

IV.3.3. Boucherot's Formula

Equation (IV.2) gives, in RMS value:

$$\underline{U_1} = \underline{E_1} = \omega N_1 \underline{\phi} = 2\pi f \cdot N_1 \frac{\phi_{max}}{\sqrt{2}} = 4.44 f \cdot N_1 \phi_{max} \qquad \textbf{IV.7}$$

So :

$$U_1 = 4.44 f \cdot N_1 \cdot S \cdot B_{max} \qquad \textbf{IV.8}$$

And :

$$U_2 = 4.44 f \cdot N_2 \cdot S \cdot B_{max} \qquad \textbf{IV.9}$$

Where *U, E* (RMS values) in (*V*), *B* (Magnetic field) in Tesla (*T*) , *S* (iron core cross-section) in (m^2) et f (ffrequency) in(Hz).

IV.3.4. Relationships between currents

$$\eta = \frac{P_2}{P_1} = 1 \qquad \textbf{IV.10}$$

Since the transformer is perfect: $P_1 = P_2$; $Q_1 = Q_2$; $S_1 = S_2$; et $\phi_1 = \phi_2$.

Which implies:

$$\frac{U_2}{U_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = m \qquad \textbf{IV.11}$$

IV.4. Real Transformer

For a real transformer, we take into account the quantities and phenomena that were neglected in the study of an ideal transformer, such as losses in the windings and core, magnetic leakage, and leakage inductances.

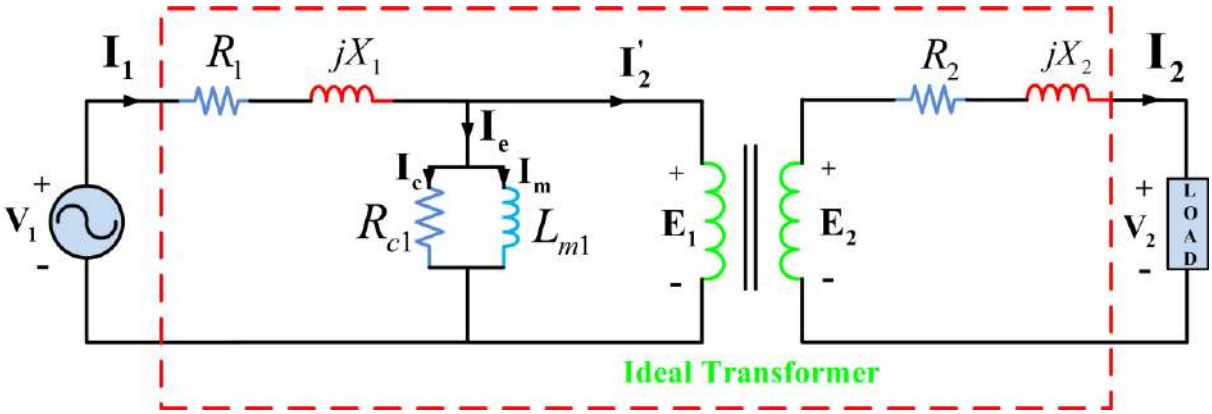


Figure IV.3. Equivalent circuit of a real transformer.

- R_1 : resistance of the primary winding;
- R_2 : resistance of the secondary winding;
- X_1 primary leakage reactance ($X_1 = L_1\omega$);
- X_2; secondary leakage reactance ($X_2 = L_2\omega$);
- R_{c1} : magnetic circuit resistance (core losses);
- L_{m1} : magnetic circuit inductance (magnetizing inductance);

IV.4.1 Relations of the Real Transformer

A. Relationships between voltages

The primary circuit equation is written as:

$$\underline{U_1} = -\underline{E_1} + R_1\underline{I_1} + jX_1\underline{I_1} \qquad \textbf{IV. 12}$$

The secondary circuit equation is written as:

$$\underline{U_2} = \underline{E_2} - R_2\underline{I_2} - jX_2\underline{I_2} \qquad \textbf{IV.13}$$

B. Relationships between currents

In a real transformer, the primary current is expressed as:

$$I_1 = I_{10} - \frac{N_2}{N_1}I_2 = I_{10} - mI_{10} \qquad \textbf{IV.14}$$

I_{10} : Current drawn by the transformer primary at no-load.

IV.5. Real Transformer in the Kapp Approximation

A. Assumption

The Kapp assumption is a commonly used simplification in the study of real transformers. It consists of assuming that:

- The no-load current I_1 (magnetizing current and core losses) is much smaller than the rated current I_1.
- Therefore, its influence can be neglected in the equivalent circuit.

This is equivalent to disconnecting the parallel branch composed of the resistance R_{c1}(core losses) and the reactance L_m (magnetizing).

B. Equivalent circuit referred to the secondary

In the Kapp approximation, the elements of the resistance R1 and the leakage reactance $X_1 = L_1\omega$, initially placed on the primary, can be referred to the secondary. This transformation is done by multiplying them by (m^2). We obtain the following circuit:

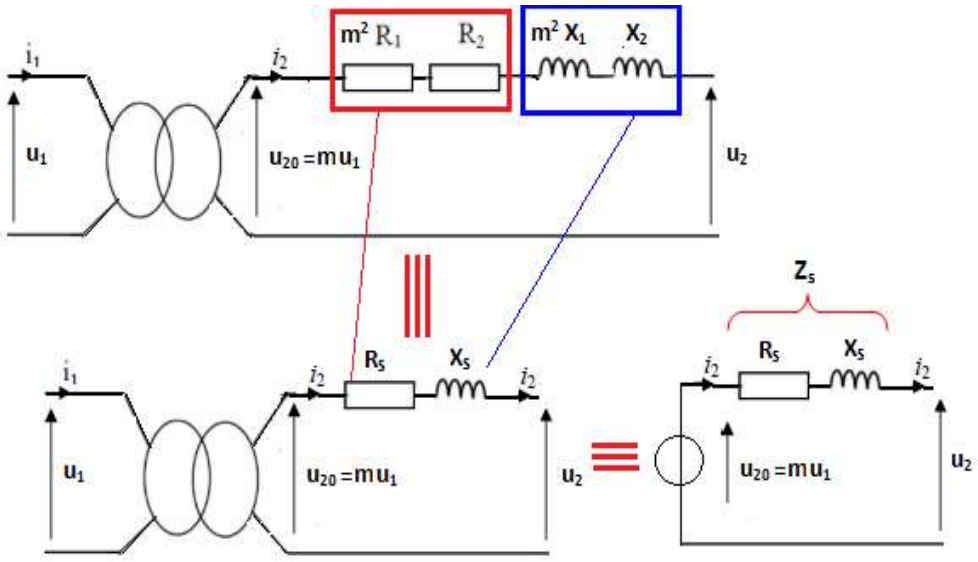


Figure IV.4. Equivalent circuit of a real transformer referred to the secondary.

$$R_S = (R_2 + m^2 R_1) \text{ et } X_S = (X_2 + m^2 X_1) \qquad \textbf{IV.15}$$

- ✓ R_S : The resistance referred to the secondary.
- ✓ X_S : The reactance referred to the secondary.

Impedance:

$$Z_S = R_S + jX_S \rightarrow Z_S = \sqrt{R_S^2 + X_S^2} \quad \textbf{IV16}$$

The secondary behaves like a voltage source:

$$U_{20} = mU_1 \quad \textbf{IV.17}$$

$$U_2 = U_{20} - Z_S I_2 = U_{20} - \Delta U_2 \quad \textbf{IV.18}$$

C. Approximate calculation of the voltage drop at the secondary

The approximate value of the voltage drop at the secondary is:

$$Z_S I_2 = \Delta U_2 \quad \textbf{IV.19}$$

The voltage drop at the secondary can then be expressed using an approximate formula, given by:

$$\Delta U_2 = R_S I_2 \cos\varphi_2 + X_S I_2 \sin\varphi_2 \quad \textbf{IV.20}$$

IV.6. Determination of Transformer Parameters and Losses

To characterize a real transformer, two experimental tests are generally performed: the no-load test and the short-circuit test. These tests allow the determination of the equivalent parameters of the Kapp model, namely

- m: le rapport de transformation ;
- Rs:The equivalent resistance of the magnetic circuit;
- Xs : The magnetizing reactance ;
- Pfer: Core losses (hysteresis and eddy currents);
- Pj : Joule losses in the windings

IV.6.1. No-Load Test

The no-load test consists of supplying the transformer on its primary winding while the secondary is open (no load connected).

- The rated voltage U1 is applied to the primary.
- With the secondary open, the current supplied by the primary is very small (magnetizing current I_0).
- The measured quantities are:
  - The no-load voltage U1,
  - The no-load current I_0,

- The power consumed P_0 (represents the no-load losses).

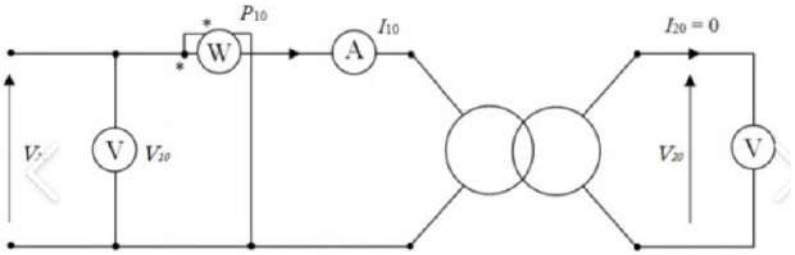


Figure IV.5. No-load test.

This test allows determining:

- ✓ **Calculation of the transformation ratio :**

$$m = \frac{U_{20}}{U_1} \qquad \textbf{IV.21}$$

- ✓ **Determination of core losses:**

$$P_{10} = R_1 I_{10}^2 + P_{fer} \qquad \textbf{IV.22}$$

At no-load, the secondary circuit is open: $I_2 = 0$ et $I_1 = I_{10} \Rightarrow P_2 = 0$ et $P_{J2} = 0$

Since I_{10} is very small, therefore $P_j = R_1 I_{10}^2$ is very small compared to P_{fer} (Joule losses are negligible compared to core losses), then we write:

$$P_{10} = P_{fer} \qquad \textbf{IV.23}$$

IV.6.2. Short-Circuit Test

The short-circuit test consists of short-circuiting the transformer's secondary winding, then supplying the primary with a reduced voltage (generally 5 to 10% of the rated voltage) so that the current reaches its rated value. The following quantities are measured:

- The supply voltage U1cc ,
- The secondary current $I_{2cc}=I_{2n}$,
- The primary power consumed P_1cc.

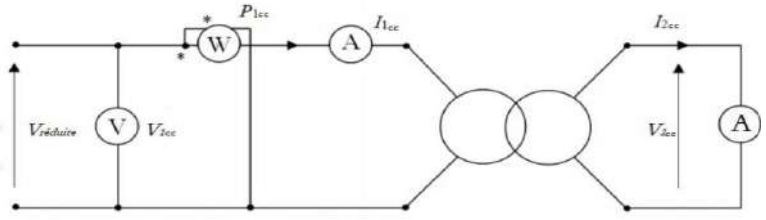


Figure IV.6. Short-circuit tests.

This test allows determining:

- ✓ **Determination of the resistance referred to the secondary Rs :**

Since the short-circuit test is carried out under rated current conditions, the measured losses correspond to the nominal losses. These are essentially due to the Joule effect in the windings and are referred to as copper losses.

Since : $U_{1CC} << U_n$ Core losses are negligible compared to Joule losses, therefore:

$\mathrm{P_{1CC}} = \mathrm{R_S I_{2CC}^2}$ Hence, the resistance Rs is:

$$\mathrm{R_S} = \frac{\mathrm{P_{1CC}}}{\mathrm{I_{2CC}^2}} \qquad \textbf{IV.24}$$

- ✓ **Reactance calculation *Xs :***

$$\mathrm{Z_S} = \frac{\mathrm{m} \cdot \mathrm{U_{1CC}}}{\mathrm{I_{2CC}}} \qquad \textbf{IV.25}$$

$$X_S = \sqrt{Z_S^2 - R_S^2} \qquad \textbf{IV.26}$$

IV.7. Transformer efficiency

The efficiency is the ratio:

$$\eta = \frac{P_2}{P_1} = \frac{P_{charge}}{P_{charge} + \sum Pertes} \qquad \textbf{IV.27}$$

$$\eta = \frac{P_2}{P_1} = \frac{U_2 I_2 \cos\varphi_2}{U_2 I_2 \cos\varphi_2 + P_{10} + R_S I_2^2} \qquad \textbf{IV.28}$$

Avec :

- P_1 : Input power ;
- P_2 : Load power;
- P_{fer} : Core loss.
- Pj: Joule loss**.**

IV.8. Three-phase transformers

Three-phase transformers are widely used in the transmission and distribution network to step up or step down the RMS voltage. Figure IV.7 shows a 250 kVA transformer used in

the distribution network. Large three-phase transformers are used for the transmission and distribution of electrical energy. They are installed in:

- Power plants;
- Interconnection substations;
- Distribution substations;
- On poles.



Figure IV.6. 250 kVA Three-Phase Transformer.

In three-phase systems, three identical single-phase transformers can be used, with their primaries and secondaries connected in either star (Y) or delta (Δ) configuration. It is also possible to place the windings on the same magnetic core.

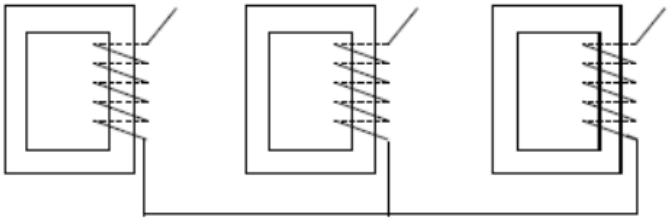


Figure IV.7. Three-phase transformer connected in star configuration.

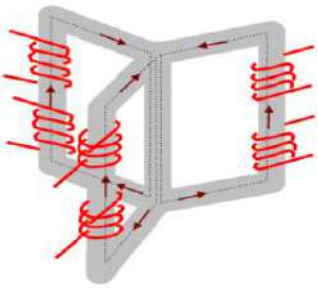


Figure IV.8. Three-phase transformer (Primary and secondary windings on the same core columns).

The primaries of these transformers will then be connected:

- Either in star (Y) and thus supplied by the phase voltages;
- Or in delta (Δ) and thus supplied by the line voltages.

Similarly, the secondary windings can be connected in star or triangle.

In this arrangement, the magnetic fluxes Φ1, Φ2, Φ3 , corresponding to completely separate magnetic circuits, are entirely independent. In contrast to the following system, this is called a three-phase transformer with independent fluxes (or open flux transformer).

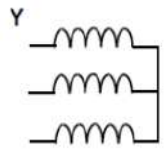
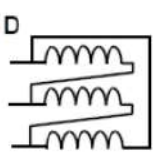
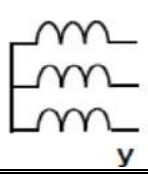
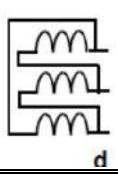
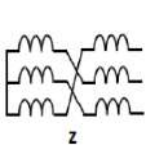
IV.8.1. Winding Connections

In addition to the usual star (Y) and delta (Δ) connections, there is the "zigzag" configuration. In this setup, the winding (primary or secondary) is divided into two identical half-windings. Each phase is then formed by connecting in series the two half-coils taken from adjacent core columns.

By convention, we designate:

- The terminals of the high-voltage primary windings are designated by capital letters: *A B C.*
- The terminals of the low-voltage secondary windings are designated by lowercase letters: *a b c*
- All quantities on the high-voltage side are written in uppercase: tension U_{AB} .
- All quantities on the low-voltage side are written in lowercase: tension U_{ab} .

Table IV.1. Winding Connections

1st letter (uppercase) – Primary connection		**2nd letter (lowercase) – Secondary connection**			**3rd letter**	**Clock-hour designation**
Star	**Delta**	**Star**	**Delta**	**Zig zag**		
Y	*D*	*y*	*d*	*z*	*N ou n*	0,1,2,....,11
						

This results in six (6) possible connections between primary and secondary:

- ✓ Y-y : star – star
- ✓ Y-d : star – delta
- ✓ Y-z : star – zigzag

- ✓ D-y : delta – star
- ✓ D-d : delta – delta
- ✓ D-z : delta – zigzag

IV.8.2. Transformation Ratio

The transformation ratio is obtained during the open-circuit test. The transformation ratio is defined as:

$$m = \frac{U_{ab}}{U_{AB}} \qquad \textbf{IV. 29}$$

This ratio does not depend solely on the ratio of the secondary voltage to the primary voltage $\frac{N_2}{N_1}$ But also on the connections chosen for the primary and secondary.

Table IV.2. Transformation Ratio

Primary / Secondary	Star	Delta
Star	$\frac{N_2}{N_1}$	$\sqrt{3}\frac{N_2}{N_1}$
Delta	$\frac{1}{\sqrt{3}}\frac{N_2}{N_1}$	$\frac{N_2}{N_1}$
Zigzag	$\sqrt{3}\frac{N_2}{N_1}$	$3\frac{N_2}{N_1}$

IV.8.3. Clock-hour Designation

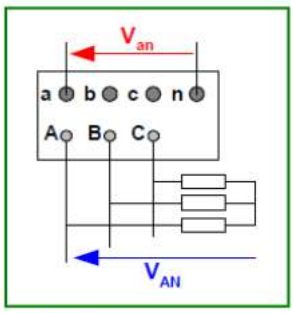
In single-phase systems, the secondary voltage could only be in phase or in opposition with the primary voltage; the corresponding terminals being marked with a dot.
In three-phase systems, the situation is more complex. A complete phasor diagram (Fresnel diagram) must be drawn to determine the phase shift between a primary voltage and a secondary voltage.

Two voltages are said to be homologous when they represent the same quantity, one on the primary and the other on the secondary. (Only the P and S differ U_{12S} et U_{12P} are homologous, just like V_{3S} et V_{3P}). Phase shifts are expressed between two homologous voltages and are always a multiple of $\pi/6$, which gives 12 possibilities.

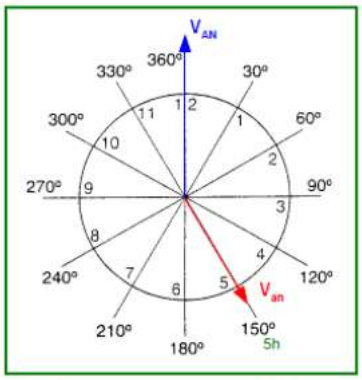
Let θ\thetaθ denote the lag of a low-voltage voltage (usually on the secondary side) relative to its high-voltage counterpart. This lag can range from 0 to 2π (a 60° lead is equivalent to a 300° lag. The clock-hour designation is the ratio of θ to $\pi/6$. The clock-hour designation I_h is:

$$I_h = \frac{\theta}{30°} \qquad \textbf{IV.30}$$

The term "clock-hour" refers to the hour that would be indicated by a clock face, where the long hand represents a high-voltage side voltage pointing at 12, and the short hand represents the homologous voltage.



D-y coupling

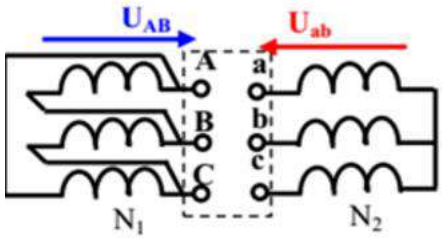


Example of 5-hour coupling

$\theta = 150°$ $(5 \times 30° = 150°)$

Figure IV.9. Clock-hour designation of a three-phase transformer.

✓ **Determination of the clock-hour designation from the transformer connection diagram:**



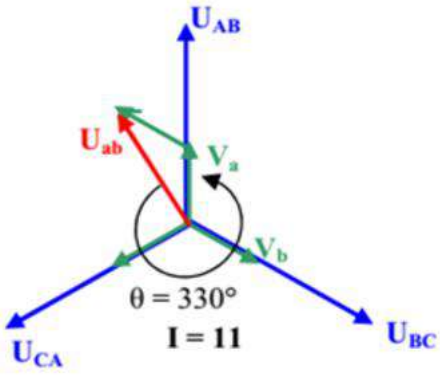


Figure IV.10. Determination of the clock-hour designation.

Thus, the designation of this transformer is Δyn11.

- *D ou*Δ : primary winding connection of the transformer, i.e., Delta;
- *y* : secondary winding connection of the transformer, i.e., Star;
- *n* : secondary neutral;
- 11 : clock-hour designation, i.e., 11 × 30° = 330° phase shift between the primary and secondary voltage.

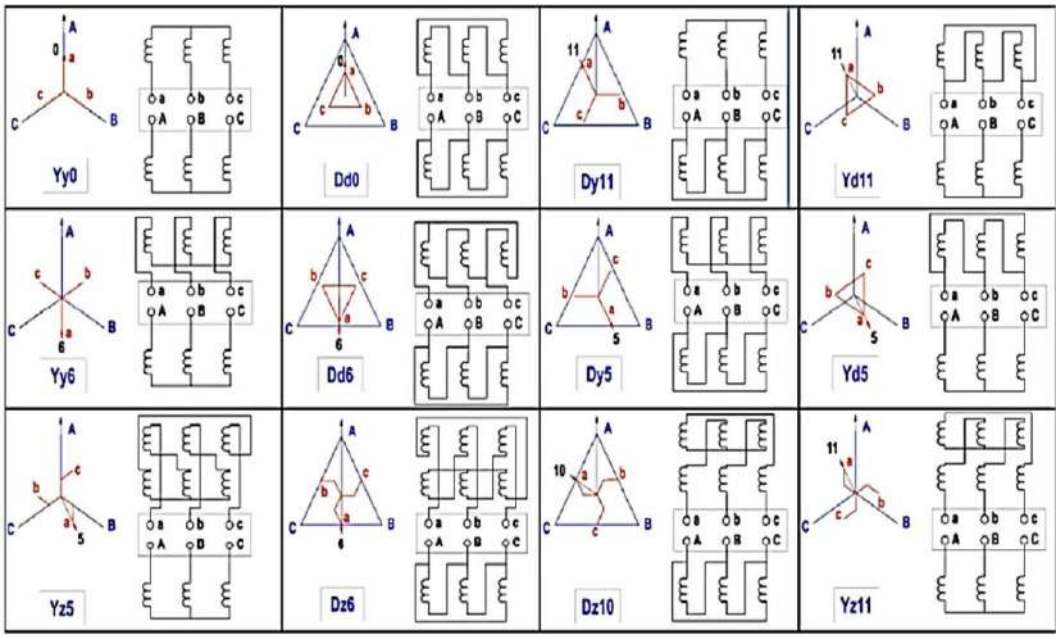


Figure IV.11. Main Connections of Three-Phase Transformers.

IV.8.4. Common Connections of Three-Phase Transformers

Figure IV.11 shows the symbol, the phase shift θ\thetaθ of Va lagging VA, the transformation ratio, the electrical connection of the phases, and the vector diagram corresponding to common transformers. The symbol includes three indications: the primary phase connection, the secondary phase connection, and the clock-hour designation.

IV.8.5. Nameplate of a Three-Phase Transformer

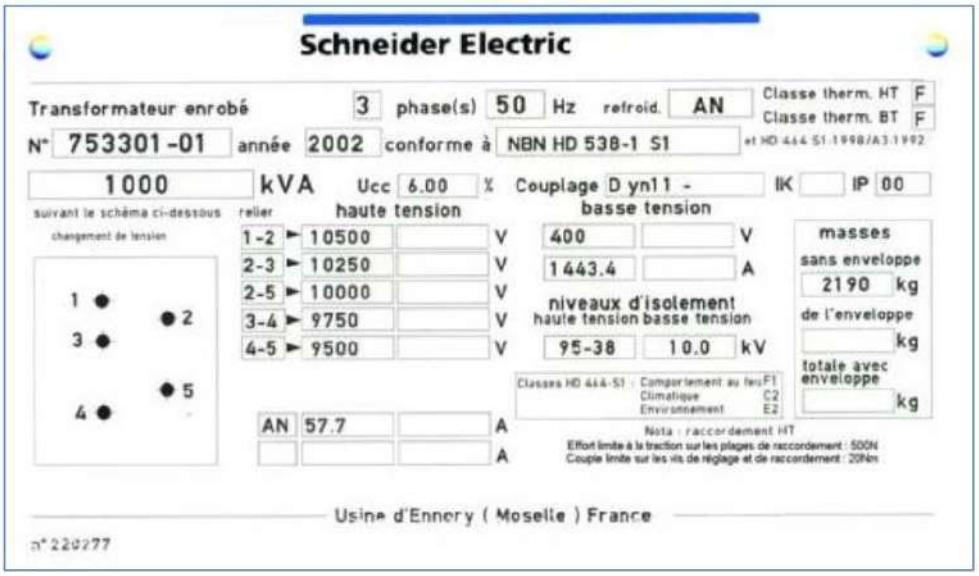


Figure IV.12. Nameplate of a Three-Phase Transformer.

- **Number of transformer phases**: 3 Phase(s)
- **Rated operating frequency of the transformer**: **50 Hz**
- **Transformer cooling type:** Cooling AN
- **ONAN:** Natural Oil (ON) circulation + Natural Air (AN) circulation
- **ONAF:** Natural Oil (ON) circulation + Forced Air (AF) circulation
- **Thermal class:** Insulation thermal class, defining the maximum temperature the winding insulation can withstand
- **Serial number**: N° 753301-01
- **Year of manufacture**: 2002
- **Rated apparent power:** 1000 kVA
- **Short-circuit voltage:** Ucc : 6 %
- **Transformer winding connection:** Dyn11
- **Impact protection rating**: IK
- **Ingress protection rating**: IP 00
- **High voltage:** Rated primary voltage values
- **Low voltage:** Rated secondary voltage and current values
- **Insulation level:** Maximum voltage between windings and ground, and between windings.

IV.8.6. Parallel Connection of Transformers

Purpose: :

- To meet an increasing demand for electrical energy;
- The primary and secondary currents then reach excessive values, causing abnormal heating of the windings and an exaggerated drop in secondary voltage.

Conditions to be met for paralleling transformers

In order to connect transformers in parallel, the following conditions must be met:

- Their primaries are supplied by the same network and their secondaries are connected to the same line;

- Their clock-hour designations are identical or belong to the same clock-hour group;
- Their power ratings differ by less than a factor of 2 (< 2) ;
- Their transformation ratios are the same;
- Their short-circuit voltages are equal (or within ±10%)..

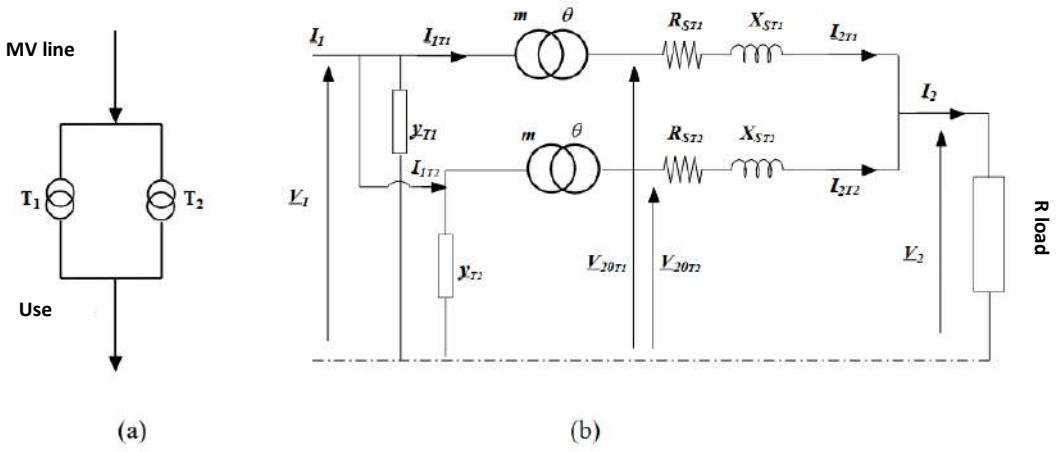


Figure IV.13. Paralleling Two Transformers.

(a) Block Diagram; (b) Operating Principle

IV.9. Main Types of Transformers

IV.9.1. Current Transformer (CT)

The current transformer (CT) is a device designed to measure high electrical currents. It provides the adaptation between the high current flowing in a circuit (which can reach several thousand amperes) and measuring devices, such as ammeters or wattmeters, as well as protection relays, which are designed for currents on the order of amperes only (Figure IV.14). The essential characteristic of a current transformer is its transformation ratio, usually expressed in the form 100 A / 5 A.

- **Principle Diagram**

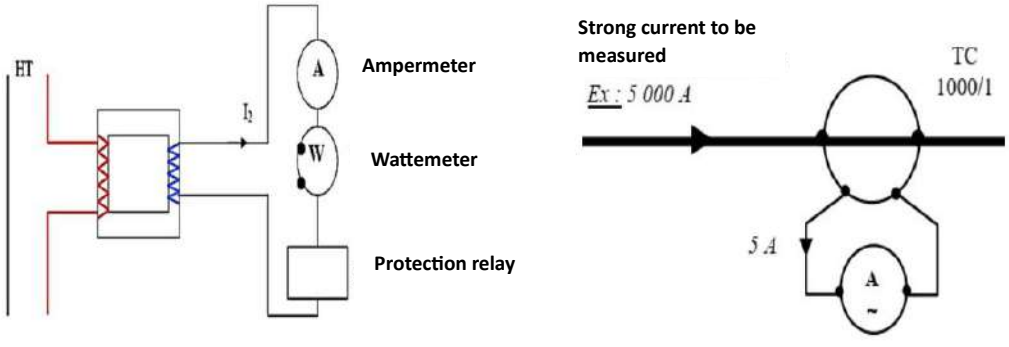


Figure IV.14. Principle diagram of current transformers.

The current transformer (CT) is generally used to obtain, from a high-intensity primary current I_1, a secondary current I_2 that is proportional to I_1 and in phase with it, but of lower magnitude. The primary winding with N1 turns is placed in series with the line conductor carrying the current I1 to be measured. If the magnetizing ampere-turns are neglected, the current I_2 flowing through the secondary winding of N2 turns generates an MMF that compensates the primary MMF:

$$N_1 \cdot I_1 = N_2 \cdot I_2 \rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad \textbf{IV.31}$$

Current transformers allow high industrial currents to be reduced to a compatible value, usually 5 amperes.

- **Symbol**

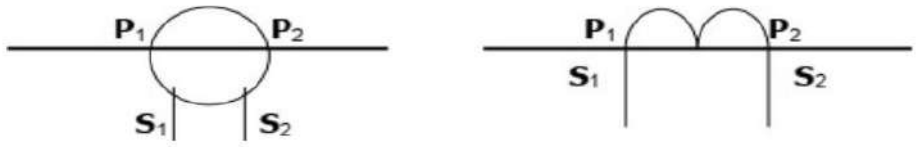


Figure IV.15. Electrical symbol of current transformers.

IV.9.2. Voltage Transformers (VTs)

When it is necessary to measure a voltage that is too high to be applied directly to a voltmeter, a step-down transformer, called a voltage transformer (VT), is used to provide a proportional representation of this voltage. The VT should only be connected to a voltmeter, whose internal impedance is very high, in order to avoid any overload. As a crucial component in the measurement chain, its design must be particularly precise.

- The primary winding is connected directly to the terminals of the voltage to be measured.
- The secondary winding is connected in parallel to the measuring devices.

- **Schéma de principe**

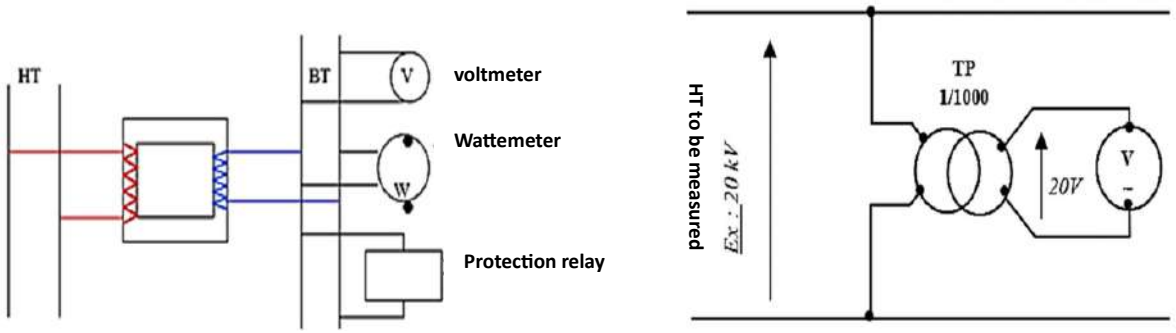


Figure IV.16. Principle diagram of voltage transformers.

Its operating principle (the same as a power transformer) under normal operating conditions:

$$\frac{U_2}{U_1} = \frac{N_2}{N_1} \qquad \textbf{IV. 32}$$

- **Symbole**

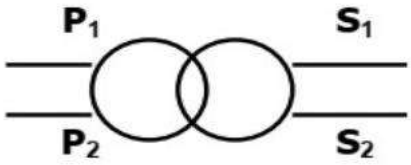


Figure IV.17. Electrical symbol of voltage transformers.

IV.9.3. On-Load Tap-Changing Transformers (OLTC)

High-voltage/high-medium-voltage (HV/MV) transformers supply the MV feeders via busbars (half-ramps). They are equipped with an on-load tap changer (OLTC) that allows the transformation ratio to be adjusted in 1% steps by changing the number of turns in the primary winding according to load variations and upstream voltage fluctuations, all without interrupting the power supply (Figure IV.18).

The operation of this on-load tap changer is controlled by a local voltage regulation controller. It adjusts the transformation ratio to maintain the MV busbar voltage around a specified setpoint, thereby contributing to voltage regulation on the MV and LV networks.

The MV voltage adjustment can be performed with or without compounding. This technique involves taking into account, by measuring the current in the transformer (and thus the average load condition of the network), the voltage drops in the downstream network. It ensures that the voltage is maintained at a point different from the source substation, such as the network's center of gravity, a priority node, or a critical point.

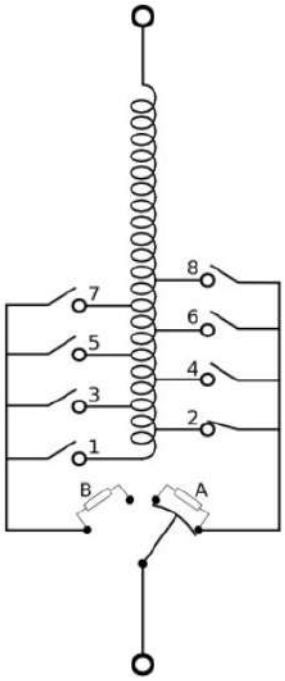


Figure IV.18. On-Load Tap Changer (OLTC).

IV.9.4. Three-Winding Transformer

This transformer has three windings on its magnetic core: a primary winding, a secondary winding, and a tertiary winding. It allows supplying two respective voltage networks from a single voltage network using a single device. Its equivalent circuit is shown in Figure IV.19:

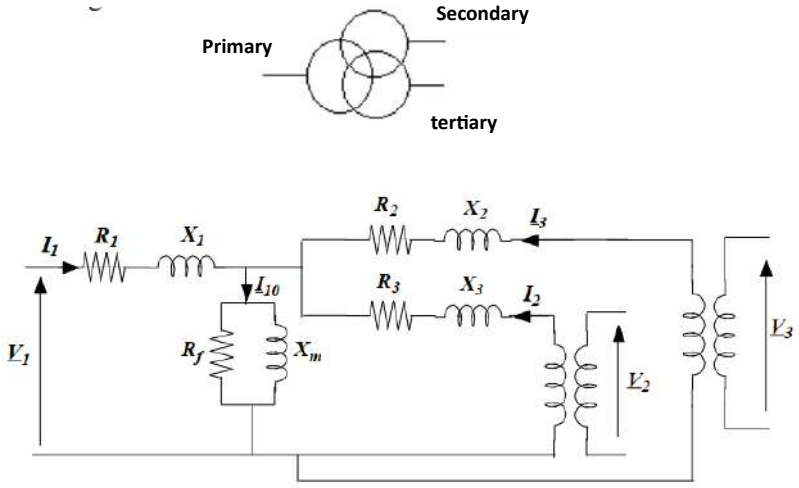


Figure IV.19. Equivalent circuit of a three-winding transformer.

If Z_{12}, Z_{23} et Z_{31} These are the impedances measured during tests by short-circuiting the primary and secondary, the secondary and tertiary, and the tertiary and primary, with the third winding each time left open. The following system of equations can be written:

$$\begin{cases} \bar{Z}_{12} = \bar{Z}_1 + \bar{Z}_2 \\ \bar{Z}_{23} = \bar{Z}_2 + \bar{Z}_3 \\ \bar{Z}_{31} = \bar{Z}_1 + \bar{Z}_3 \end{cases} \Rightarrow \begin{matrix} \bar{Z}_1 = \frac{\bar{Z}_{12}+\bar{Z}_{13}-\bar{Z}_{23}}{2} \\ \bar{Z}_2 = \frac{\bar{Z}_{21}+\bar{Z}_{23}-\bar{Z}_{13}}{2} \\ \bar{Z}_3 = \frac{\bar{Z}_{31}+\bar{Z}_{32}-\bar{Z}_{12}}{2} \end{matrix} \qquad \textbf{IV.33}$$

IV.9.5. Phase-Shifting Transformer

The phase-shifting transformer is a special type of power transformer, mainly used to regulate the flow of active power between multiple three-phase power lines. In an AC transmission line, the active power transferred is proportional to the sine of the power angle. Thus, when two lines operate in parallel, adjusting this angle allows controlling the distribution of currents between them, optimizing transmission capacity, and preventing the risk of overload.

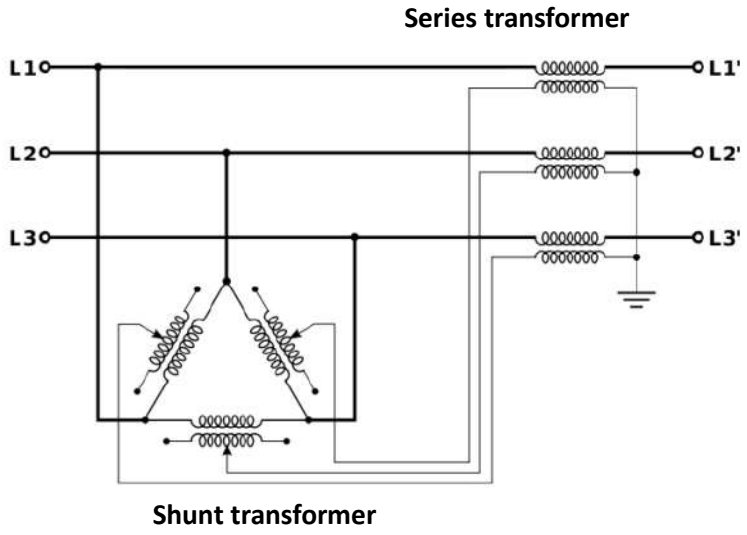
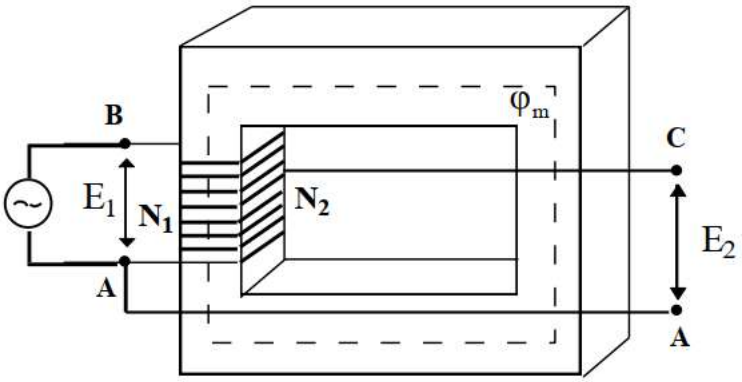


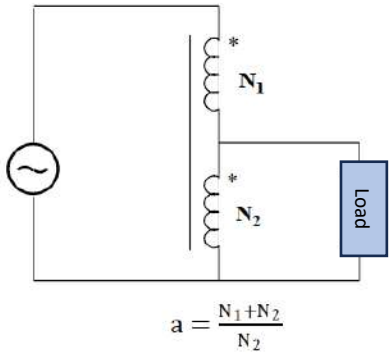
Figure IV.20. Equivalent circuit of a phase-shifting transformer.

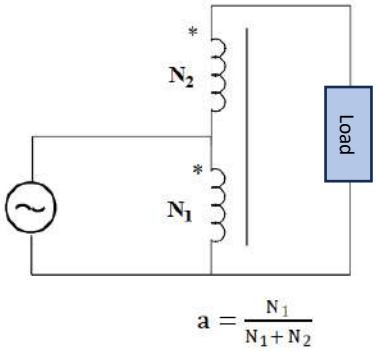
IV.9.6. Autotransformer

It is a static transformer in which the primary and secondary windings are no longer electrically separate. It has a single winding, with the secondary using a portion of the primary turns (N1 turns between terminals A and B, and N2 turns between terminals A and C). There are two possible configurations: step-down and step-up (Figure IV.21):



(a) Electrical and magnetic circuit





(b) Step-down transformer (c) Step-up transformer

Figure IV.21. Autotransformer.

✓ **Advantages:**

- Only one winding is required, and the wire for the common portion can be thinner;
- Joule (RI^2) losses are reduced, resulting in higher efficiency.

✓ **Disadvantage:**

The primary and secondary windings are no longer electrically isolated. In case of a fault, high voltage may propagate to the low-voltage side.

✓ **Usage:** Due to the hazards associated with the lack of electrical isolation, the autotransformer is generally not used in distribution networks. In some countries, it is

employed in medium voltage (MV) systems when voltages are close (e.g., 15/20 kV) and in extra-high voltage (EHV) substations for interconnection purposes.

IV.10. Système d'unité relative (PU)

The per-unit (PU) system is a system of normalized quantities that allows all electrical network elements (generators, transformers, transmission lines, etc.) to be expressed with reference to a single standard. This results in reduced or per-unit values.The per-unit system was originally developed to simplify manual calculations. Despite the widespread use of modern computational tools, it remains extensively used. Its main advantage lies in the ability to quickly detect calculation errors in power system analysis. Indeed, quantities expressed in per-unit typically fall within a limited range, making it easy to identify any abnormal value that deviates from the expected range.

A. Basic formula:

The per-unit value of any quantity is defined as the ratio of that quantity to its corresponding base value. Real quantities such as voltage (V), current (I), active power (P), reactive power (Q), apparent power (S), resistance (R), reactance (X), and impedance (Z) can be expressed in per-unit (pu) or as a percentage (%) as follows:

$$Q\text{uantity (pu)} = \frac{\text{Actual quantity}}{\text{Base quantity}} \qquad \textbf{IV.34}$$

$$\text{Quantity (\%)} = \text{Quantity (pu)} \cdot 100\% \qquad \textbf{IV.35}$$

For example :

$$S_{pu} = \frac{S_{Actua}}{S_{base}} \qquad \textbf{IV.36}$$

$$V_{pu} = \frac{V_{Actual}}{V_{base}} \qquad \textbf{IV.37}$$

$$I_{pu} = \frac{I_{Actual}}{I_{base}} \qquad \textbf{IV.38}$$

$$Z_{pu} = \frac{Z_{Actual}}{Z_{base}} \quad \textbf{IV.39}$$

Generally, the three-phase apparent base power S_{base} and the composite voltage V_{base} are selected. The base current and base impedance are then dependent on S_{base} and V_{base} and must obey the laws of circuitry. This is given by:

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} \quad \textbf{IV.40}$$

$$Z_{base} = \frac{V_{base}/\sqrt{3}}{I_{base}} \quad \textbf{IV.41}$$

By replacing I_{base} with Z_{base}, the base impedance becomes:

$$Z_{base} = \frac{V_{base}^2}{S_{base}} \quad \textbf{IV.42}$$

$$R_{base} = X_{base} = Z_{base} = \frac{1}{Y_{base}} \quad \textbf{IV.43}$$

The phase and line quantities expressed per unit are the same, the laws of electricity are valid, i.e.:

$$S_{pu} = V_{pu} \cdot I_{pu}^* = P_{pu} + jQ_{pu} \quad \textbf{IV.44}$$

$$V_{pu} = Z_{pu} \cdot I_{pu} \quad \textbf{IV.45}$$

B. Change of base :

Generally, the impedance values of generators and transformers provided by manufacturers are given in a per-unit (pu) system, where the base quantities correspond to the rated voltage and rated power of the device.

The problem that then arises is how to standardize the data, that is, to convert the impedances and admittances—expressed in any given system—into the system associated with the chosen base quantities (S_{base} et V_{base}) chosen for the section under consideration.

We can write, for two different base systems:

$$Z = Z_{pu1} \cdot Z_{base1} = Z_{pu2} \cdot Z_{base2} \qquad \textbf{IV.46}$$

Where :

$$Z_{pu2} = Z_{pu1} \cdot \frac{Z_{base1}}{Z_{base2}} = Z_{pu1} \cdot \left(\frac{U_{base1}}{U_{base2}}\right)^2 \left(\frac{S_{base2}}{S_{base1}}\right) \qquad \textbf{IV.47}$$

For admittances, we obtain a similar formula:

$$Y_{pu2} = Y_{pu1} \cdot \frac{Y_{base1}}{Y_{base2}} = Y_{pu1} \cdot \left(\frac{Y_{base2}}{Y_{base1}}\right)^2 \left(\frac{S_{base1}}{S_{base2}}\right) \qquad \textbf{IV.48}$$

Chapter V

Short-circuit current calculation

V.1. Introduction

Electrical networks, although designed to operate reliably and continuously, remain exposed to various incidents that can disrupt their operation. Among these incidents, short circuits are the most frequent and dangerous faults. They typically occur when one or more conductors accidentally come into contact with each other or with the ground, resulting in the flow of a high-intensity current.

Short circuits can be caused by several factors, such as lightning, adverse weather conditions, deterioration of equipment insulation, or operational errors. The magnitude of the short-circuit current mainly depends on the network impedance at the fault location and can reach values several times higher than the nominal current. These high currents can cause significant damage to equipment, lead to power outages, and endanger human safety.

V.2. Importance of Short-Circuit Current Calculation

The calculation of short-circuit currents is a fundamental step in the design, operation, and protection of electrical networks. Two values of the short-circuit current must be known:

- ✓ **Maximum short-circuit current:**

  - Determines the breaking capacity of circuit breakers.
  - Allows verification of the closing capacity of devices.
  - Used to check the electrodynamic and thermal withstand of cables, busbars, and switchgear.
  - Must be calculated accurately to prevent serious damage to equipment.

- ✓ **Minimum short-circuit current :**

  - Essential for selecting the tripping curve of circuit breakers and fuses.
  - Important when cable lengths are long or the source is impedance-limited (e.g., generators, inverters).
  - Ensures the protection of personnel, particularly in TN and IT neutral grounding schemes, where automatic disconnection must be guaranteed.

V.3. Causes of Short-Circuit Currents

Short circuits in electrical networks can have multiple origins, related to environmental conditions as well as technical or human factors. The main causes include:

- **Atmospheric conditions:** Phenomena such as lightning, storms, or strong winds can cause temporary overvoltages or partially damage equipment, leading to dielectric breakdowns and thus short circuits.
- **Aging of equipment:** Over time, materials lose their mechanical and insulating properties, which can result in ruptures or local failures responsible for electrical faults.
- **Insulation defects:** A degraded or damaged insulator can no longer perform its function, promoting contact between conductors, between phases, or even between turns in a transformer.
- **Human errors:** Improper operation or forgetting to ground equipment during maintenance can cause serious short circuits. These errors can also generate temporary overvoltages capable of causing dielectric breakdown.
- **Acts of vandalism:** For example, intentionally throwing metallic objects onto an overhead line can create accidental contact between conductors.
- **Animals:** Birds can cause short circuits. For instance, the large wingspan of a stork in flight can simultaneously contact two conductors.

V.4. Consequences of Short-Circuit Faults

They vary depending on the nature and duration of the faults, the location within the installation, and the magnitude of the current:

- **At the fault location:**
  - Formation of electric arcs causing degradation or destruction of insulation.
  - Partial or total melting of conductors.
  - Fire hazards and risk to personnel nearby.

- **On the affected circuit:**
  - Occurrence of significant electrodynamic forces that can cause deformation of busbars and tearing of cables.

- Excessive heating due to increased Joule losses, leading to accelerated deterioration of insulation.

- **On the rest of the network:**
  - Voltage dips during the fault clearing time (from a few milliseconds to several hundred milliseconds).
  - Partial or total shutdown of the network depending on its configuration and the selectivity of protections.
  - Risks of dynamic instability and loss of synchronism of synchronous machines.
  - Disturbances in control and command circuits.

V.5. Main Types of Short-Circuit Faults

In an electrical network, several types of faults can occur. The most common are:

A. Single-Phase-to-Ground Short Circuit

- It is the most common fault.
- It occurs when a phase accidentally comes into contact with the ground.

B. Two-Phase Short Circuit

- Two phases come into direct contact, without connection to the ground.
- It generates high fault currents and significant imbalances.

C. Two-Phase-to-Ground Short Circuit

- Two phases simultaneously come into contact with the ground.
- This type of fault is more severe than a single-phase fault, as it involves two conductors at the same time.

D. Three-Phase Short Circuit

- All three phases are short-circuited with each other, with or without connection to the ground.
- It is the most severe fault because it results in very high currents and a sudden voltage drop.

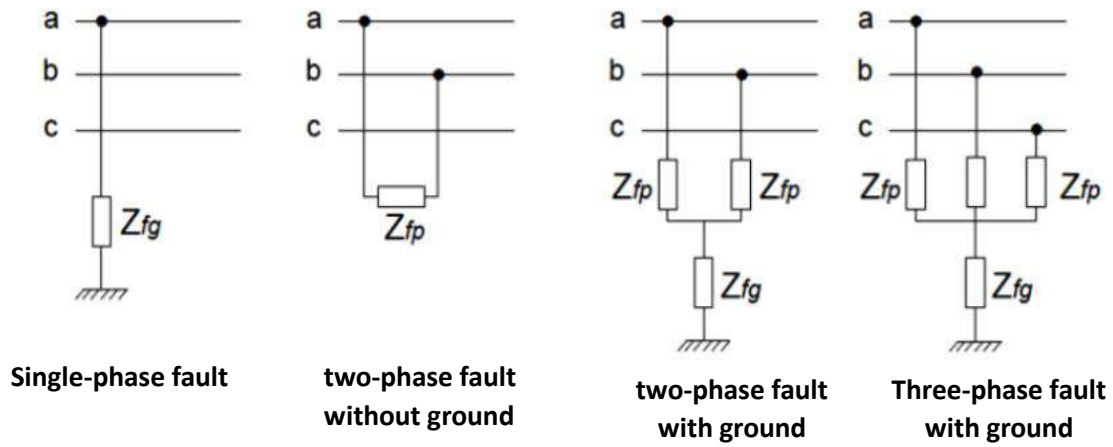


Figure V.1. Different Types of Fault.

V.6. Characteristics of Short Circuits

Short-circuit faults can be classified according to several criteria:

✓ **According to their duration :**

- **Self-extinguishing:** disappear on their own after a short time (e.g., arc ignition followed by natural extinction).
- **Transient:** persist for a limited time before disappearing.
- **Permanent:** require the intervention of a protection device to be cleared.

✓ **According to their origin:**

- Mechanical: rupture of a conductor or accidental contact between two conductors (caused by a metallic object, falling tools, or the presence of animals).
- Other possible causes: aging of insulation, atmospheric phenomena (lightning, wind, pollution), or human errors.

✓ **According to their location:**

- Internal: when they occur inside an electrical machine (motor, transformer) or a switchgear panel.
- External: when they occur on lines or in distribution networks.

In addition to these characteristics, short circuits can be:

- **Single-phase:** 80% of cases;
- **Two-phase:** 15% of cases. These faults often evolve into three-phase faults;
- **Three-phase:** only 5% of cases from the outset.

V.7. Determination of Short-Circuit Current

A simplified network can be reduced to a constant AC voltage source, a switch, and an impedance Zcc representing all the impedances upstream of the switch, along with a load impedance Zs (Figure V.2).

In reality, the source impedance comprises everything upstream of the short-circuit, with different voltage levels (HV, LV) and series lines with varying cross-sections and lengths. In the diagram of Figure V.2, with the switch closed, the service current Is flows through the network.

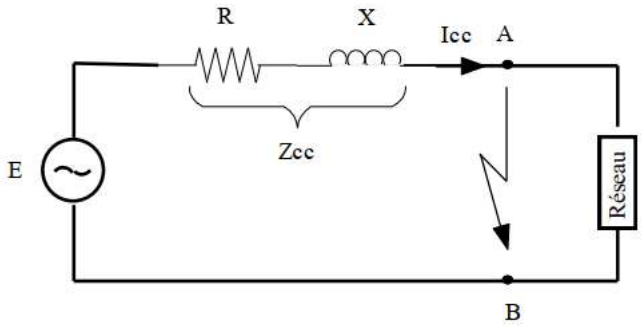


Figure V.2. Equivalent Diagram of the Upstream Network During a Short Circuit.

A fault with negligible impedance occurring between points A and B generates a very high short-circuit current Icc, limited only by the impedance Zcc . The current Icc is established according to a transient regime, depending on the reactances X and resistances R that make up the impedance Zcc:

$$Z_{CC} = \sqrt{R^2 + X^2} \qquad \textbf{V.1}$$

In power distribution, the reactance $X = L.\ \omega$ is generally much higher than the resistance RRR, and the ratio $\frac{R}{X}$ lies between 0.1 and 0.3. For these low values, it is practically equal to the short-circuit power factor cos φcc , i.e.:

$$\cos \varphi_{CC} = \frac{R}{\sqrt{R^2+X^2}} \qquad \textbf{V.2}$$

However, the transient behavior of the short-circuit current depends on the distance of the fault point from the generators. This “distance” does not necessarily refer to a geographical distance, but implies that the generator impedances are lower than the connecting impedances between them and the fault point.

V.7.1. Fault Distant from Generators

This is the most common case. The transient regime is then that resulting from the application of a voltage to a self-resistance circuit:

$$e = E\sqrt{2}\sin(\omega \cdot t + a) \qquad \textbf{V.3}$$

The intensity I of the short-circuit current consists of two terms:

$$I = I_{CA} + I_{CC} \quad \textbf{V.4}$$

- **An alternative component I_{CA} (or periodic),** with a frequency identical to that of the grid,

$$I_{CA} = I\sqrt{2}\sin(\omega \cdot t + a - \varphi) \quad \textbf{V.5}$$

Où :

$$I = \frac{E}{Z_{CC}} \quad \textbf{V.6}$$

α : angle électrique qui caractérise le décalage entre l'instant initial du défaut et l'origine de l'onde de tension.

- **A continuous (or aperiodic) I_{CC} component,** which gradually decreases over time due to the effect of the circuit's inductance and resistance.

$$I_{CC} = -I\sqrt{2}\sin(a - \varphi)e^{-\frac{R}{L}t} \quad \textbf{V.7}$$

Its initial value depends on α, and its damping is faster when R/L is high. At the initial instant of the short circuit, I is zero by definition (the service current Is being neglected), hence:

$$I = I_{CA} + I_{CC} = 0 \quad \textbf{V.8}$$

Figure V.3 shows the graphical construction of iii by the algebraic addition of the ordinates of its two components ia (I_{CA}) and ic(I_{CC}).

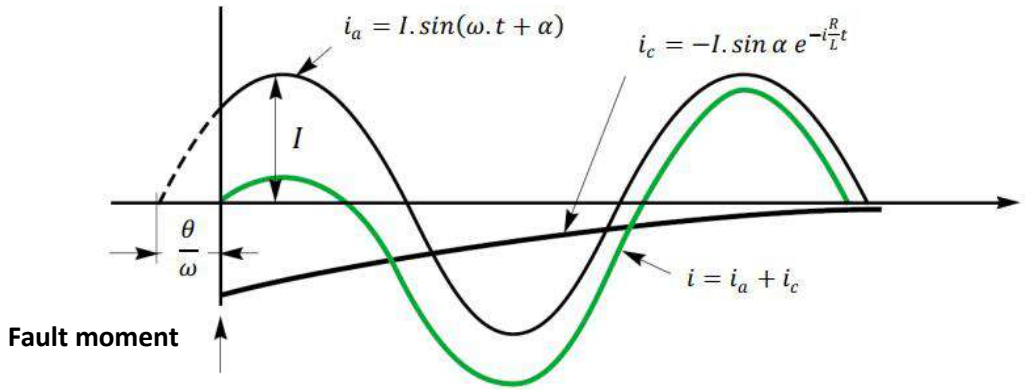


Figure V.3. Graphical Representation and Decomposition of a Short-Circuit Current Established at a Point Distant from a Generator.

V.7.2. Fault Close to Generators

When a short circuit occurs very close to a generator, the machine's internal impedance dominates the circuit. Its variation leads to a gradual damping of the short-circuit current.

In practice, the transient regime is complex, as the generator's electromotive force (EMF) also varies under the effect of the fault. To simplify the study, it is generally assumed that the EMF remains constant, while the machine's internal reactance evolves through three successive stages:

- Subtransient regime: lasts about 10 to 20 ms after the fault occurs. The reactance is very low, resulting in a very high short-circuit current.
- Transient regime: can last up to about 500 ms. The reactance increases, gradually reducing the fault current.
- Steady-state (or synchronous) regime: corresponds to the synchronous reactance of the machine. The current stabilizes at a lower and constant value.

The subtransient reactance is the lowest, followed by the transient reactance, and then the steady-state reactance, which is the highest. This successive evolution of reactances leads to a gradual decrease in the short-circuit current magnitude.
The total short-circuit current can be considered as the sum of four components:

- **Three alternating components**:
  - The subtransient component, dominant immediately after the fault occurs,
  - The transient component, which persists for a few hundred milliseconds,
  - The steady-state component, corresponding to the established regime with the synchronous reactance.

- **One direct (DC) component**, Results from the establishment of current in the inductive circuit and decays exponentially over time.

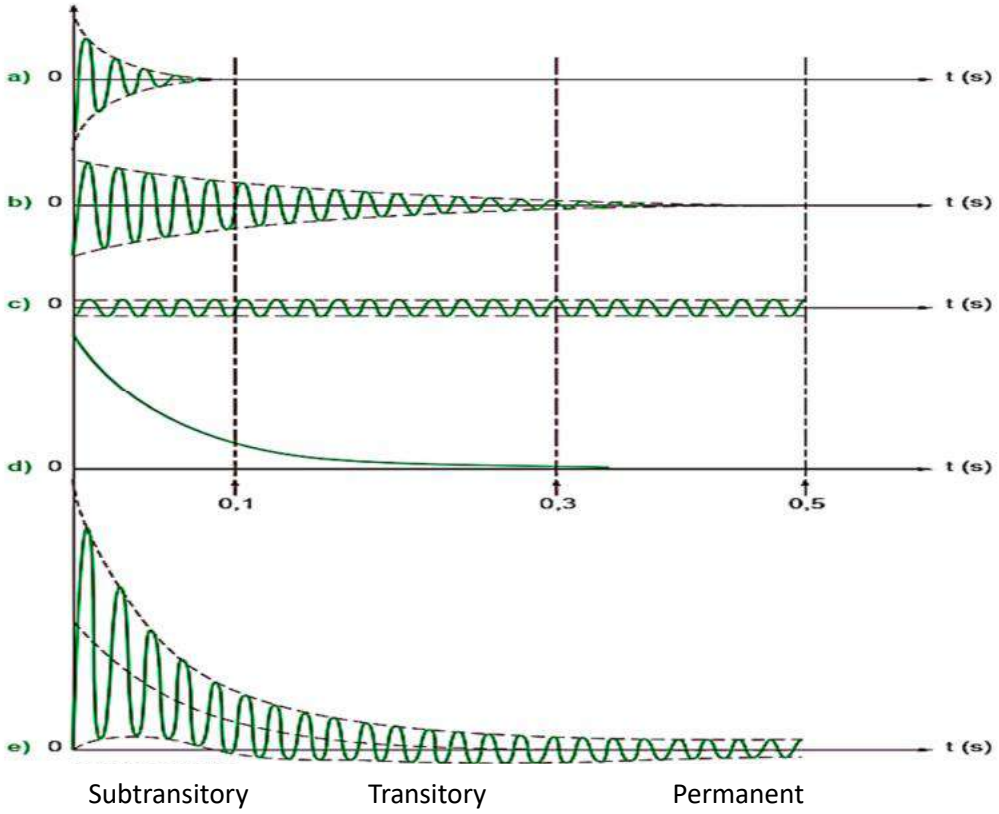


Figure V.4. Form of the total short-circuit current Icc, curve (e), with the contribution of: (a) the subtransient reactance (b) the transient reactance (c) the synchronous reactance (d) the DC component

V.8. Value of I_{CC} According to Different Types of Short-Circuits

There are mainly two types of short circuits:

- ✓ **Symmetrical short circuits:** A short circuit is called symmetrical when all three phases of the network are affected identically, meaning that the currents and voltages remain balanced and 120° apart.

  Symmetrical short circuits mainly involve three-phase faults, either isolated or grounded. They preserve the system's balance, which allows calculations to be simplified by considering only a single phase. Although rare in practice, they generally produce the highest fault currents.

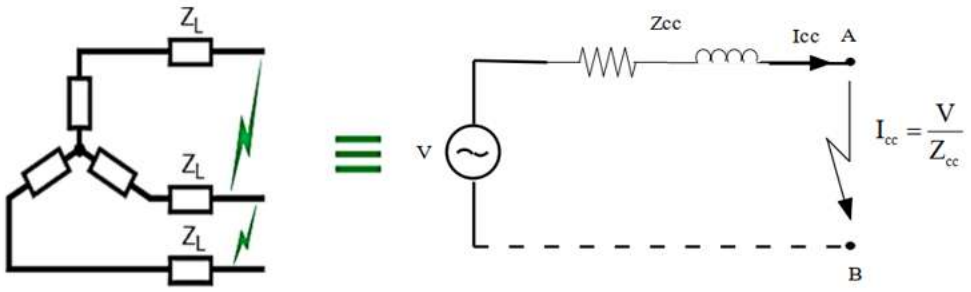
- ✓ **Asymmetrical short circuits:** When such a fault occurs, it generates asymmetrical currents, characterized by different amplitudes in the three phases as well as a non-

uniform phase shift. These faults correspond to asymmetrical short circuits, among which we distinguish:

- Single-phase-to-ground short circuit (phase-to-ground),
- Two-phase isolated short circuit (phase-to-phase),
- Two-phase-to-ground short circuit (phase-to-phase-to-ground).

V.8.1. Calculation of Symmetrical Short-Circuit Currents Using the Impedance Method

✓ **Three-phase short circuit :**



Z_{CC} : Equivalent impedance for a three-phase short circuit

Figure V.5. Court-circuit triphasé.

The short-circuit current I_{CC} is:

$$I_{CC3} = \frac{U}{\sqrt{3} \times Z_{CC}} \qquad \textbf{V.9}$$

With U (line-to-line voltage) corresponding to the transformer's no-load voltage, which is 3 to 5% higher than the voltage at full load.

For example, in 380 V networks, the adopted line-to-line voltage is U=410 V, with the corresponding phase voltage being:

$\frac{U}{\sqrt{3}} = 127.$

The calculation of the short-circuit current then boils down to calculating the impedance Zcc, the equivalent impedance of all the impedances traversed by the short-circuit current Icc, from the generator to the fault point, including the source and the lines .

$$Z_{CC} = \sqrt{(\sum R)^2 + (\sum X)^2} \qquad \mathbf{V.10}$$

Where, $\sum R$ sum of resistances in series, et $\sum X$ = sum of series reactances.

The three-phase fault is generally considered to generate the highest fault currents. Indeed, in the equivalent diagram of a polyphase system, the fault current is limited only by the impedance of a single phase under the phase voltage of the network. The calculation of Icc3 is therefore essential for determining the characteristics of the equipment, particularly the maximum currents and the electrodynamic stresses they must be able to withstand.

- ✓ **Two-Phase Isolated Short Circuit :**

It corresponds to a fault between two phases (Figure V.6), supplied under a line-to-line voltage U.

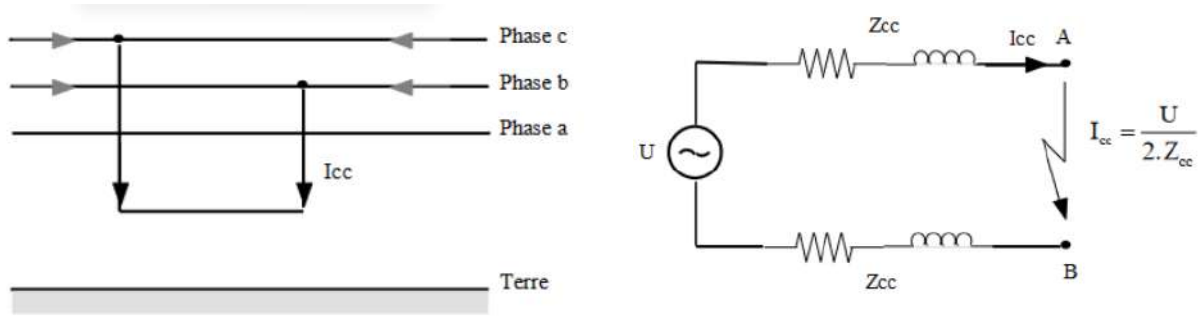


Figure V.6. Two-Phase Isolated Short Circuit.

The short-circuit current Icc2 is then lower than that of the three-phase fault:

$$I_{2CC} = \frac{U}{2Z_{CC}} = \frac{\sqrt{3}}{2} I_{CC} \approx 0.86 \qquad \mathbf{V.11}$$

- ✓ **Single-Phase Short Circuit :**

It corresponds to a fault between a phase and the neutral (Figure V.7), supplied under a phase voltage. $V = \frac{u}{\sqrt{3}}$

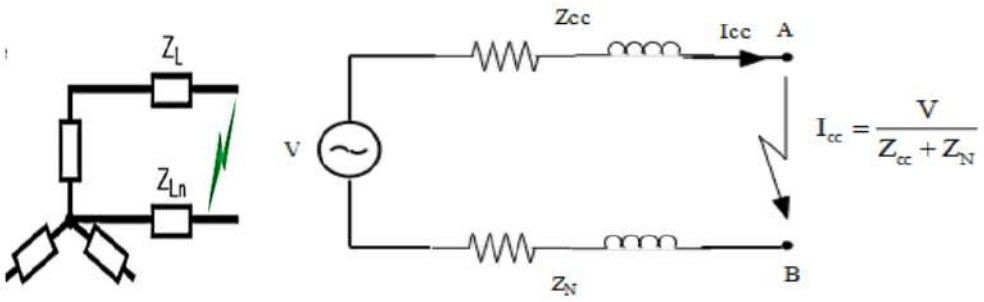


Figure V.7. Equivalent Impedance for a Single-Phase Short Circuit.

The short-circuit current Icc1 is then:

$$I_{CC} = \frac{U}{\sqrt{3}(Z_{CC} + Z_{LN})} \qquad \textbf{V.12}$$

In certain specific cases of single-phase faults, the homopolar impedance of the source may be lower than the phase-to-phase short-circuit impedance Z_{CC}, for example at the terminals of a star-zigzag connected transformer or a generator in the subtransient regime. In such situations, the single-phase fault current can even exceed that generated by a three-phase fault.

- ✓ **Ground Fault Short Circuit (Single-Phase or Two-Phase):**

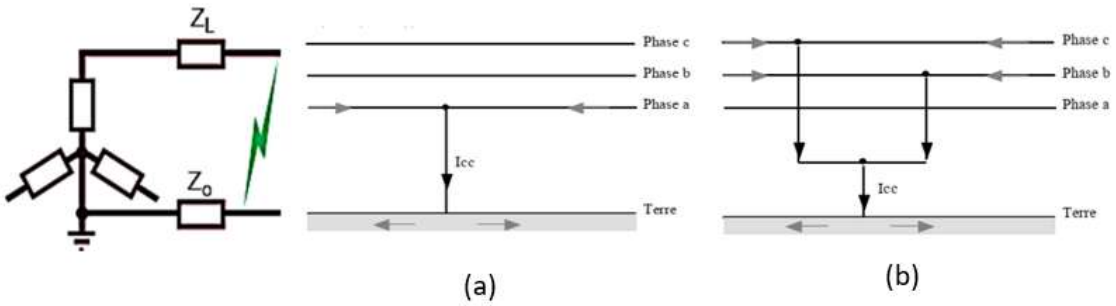


Figure V.8. Ground Short Circuit.

(a) Single-Phase-to-Ground Short Circuit

(b) Two-Phase-to-Ground Short Circuit

Its calculation may be necessary, depending on the neutral grounding scheme, for selecting the setting thresholds of homopolar protection devices (HV) or differential protection devices (LV).

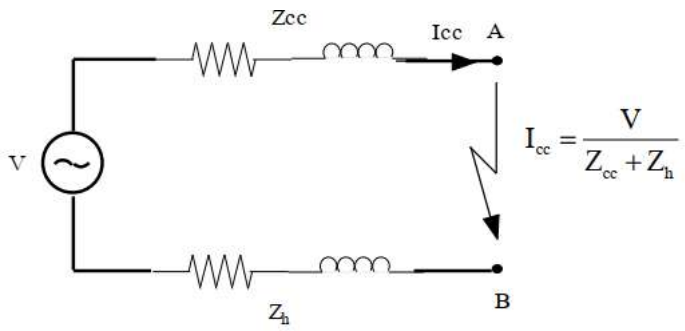


Figure V.9. Equivalent Impedance for a Two-Phase-to-Ground Short Circuit.

The short-circuit current I_{CCh} is expressed as follows:

$$I_{CCh} = \frac{\frac{U}{\sqrt{3}}}{Z_{CC} + Z_h)} \qquad \textbf{V.13}$$

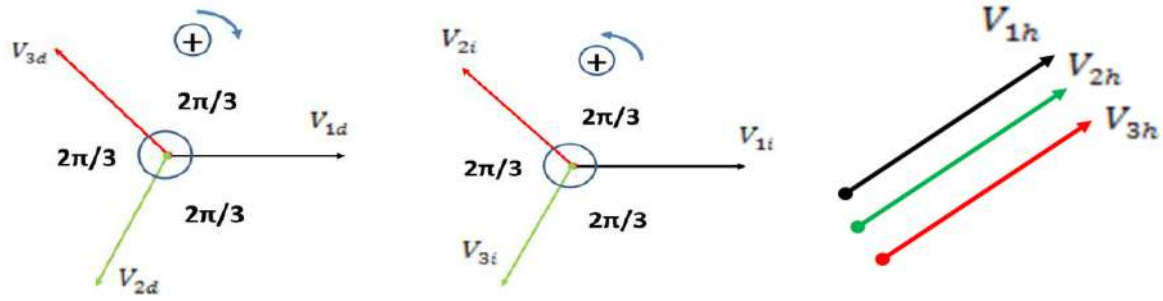
V.8.2. Calculation of Short-Circuit Currents in Radial Networks Using Symmetrical Components

The calculation of unbalanced conditions then becomes quite complex, as the traditional laws of three-phase networks are not applicable. The method of symmetrical components significantly simplifies this type of calculation. It is essentially based on the following property: any system of unbalanced sinusoidal quantities ca n be decomposed into three balanced three-phase systems, called the symmetrical components of the unbalanced system.

The method of symmetrical components is used, which consists of reducing the real system to the superposition of three independent single-phase systems, called:

- Positive-Sequence (Direct) Component => index d or 1
- Negative-Sequence Component => index i or 2
- Zero-Sequence Component => index h or 0

For each system, respectively 1, 2, 0, the voltages U^0, U^1, U^2 , and the currents I^0, I^1, I^2, and are related by the impedances Z^0, Z^1,et Z^2 of the same system.



Positive-Sequence Componen Negative-Sequence Component Zero-Sequence Component

Figure V.10. Representation of Symmetrical Components.

☞ **Opérateur a :**

« a » is a vector operator that rotates the vector to which it is applied by +2π/3.

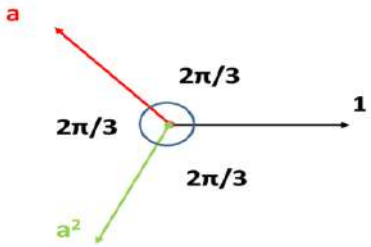


Figure V.11. Operator « a ».

$$\begin{cases} a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ a^2 = e^{j\frac{4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \\ a^3 = e^{j\frac{6\pi}{3}} = 1 \end{cases} \Rightarrow \; a + a^2 + a^3 = 1 \qquad \textbf{V.14}$$

V.8.3. Decomposition of a Three-Phase System into Symmetrical Components

✓ **Symmetrical Components of Voltage**

Let us consider a three-phase system composed of three sinusoidal quantities with the same angular frequency but arbitrary phase angles and amplitudes, represented by complex numbers.

es $\overline{V}_1$, $\overline{V}_2, \overline{V}_3$. According to Fortescue's theorem, such a system can always be decomposed into three simpler systems:

- Positive-sequence system
- Negative-sequence system
- Zero-sequence system

$$\begin{cases} \overline{V_1} = \overline{V_{1d}} + \overline{V_{1i}} + \overline{V_{1h}} \\ \overline{V_2} = \overline{V_{2d}} + \overline{V_{2i}} + \overline{V_{2h}} \\ \overline{V_3} = \overline{V_{3d}} + \overline{V_{3i}} + \overline{V_{3h}} \end{cases} \quad \textbf{V.15}$$

Let us define

$$\overline{V_{1d}} = \overline{V_d} \Rightarrow \begin{matrix} \overline{V_{1d}} = \overline{V_d} \\ \overline{V_{2d}} = a^2\overline{V_d} \\ \overline{V_{3d}} = a\,\overline{V_d} \end{matrix} \text{ et } \begin{matrix} \overline{V_{1i}} = \overline{V_i} \\ \overline{V_{2i}} = a\,\overline{V_i} \\ \overline{V_{3i}} = a^2\overline{V_i} \end{matrix} \text{ et } \overline{V_{1h}} = \overline{V_{2h}} = \overline{V_{3h}} = \overline{V_h} \quad \textbf{V.16}$$

Equation V.16 can be rewritten as follows:

$$\begin{cases} \overline{V_1} = \overline{V_d} + \overline{V_i} + \overline{V_h} \\ \overline{V_2} = a^2\overline{V_d} + a\overline{V_i} + \overline{V_h} \\ \overline{V_3} = a\overline{V_d} + \overline{a^2V_i} + \overline{V_h} \end{cases} = \begin{cases} \overline{V_1} = \overline{V_h} + \overline{V_d} + \overline{V_i} \\ \overline{V_2} = \overline{V_h} + a^2\overline{V_d} + a\overline{V_i} \\ \overline{V_3} = \overline{V_h} + a\,\overline{V_d} + a^2\overline{V_i} \end{cases} \quad \textbf{V.17}$$

In Matrix Form:

$$\begin{bmatrix} \overline{V_1} \\ \overline{V_2} \\ \overline{V_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} \text{ Avec : } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad \textbf{V.18}$$

A : This is called the symmetrical components transformation matrix (Fortescue matrix).

- **Inverse Transformation :**

$\det(A) = 3a(a - 1) \neq 0$, So A is invertible; the system can be solved as follows:

$$\begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \overline{V_1} \\ \overline{V_2} \\ \overline{V_3} \end{bmatrix} \quad \textbf{V.19}$$

$$\begin{cases} \overline{V_h} = \frac{1}{3}(\overline{V_1} + \overline{V_2} + \overline{V_3}) \\ \overline{V_d} = \frac{1}{3}(\overline{V_1} + a\overline{V_2} + a^2\overline{V_3}) \\ \overline{V_i} = \frac{1}{3}(\overline{V_1} + a^2\overline{V_2} + a\overline{V_3}) \end{cases} \quad \textbf{V.20}$$

$\overline{V_h}, \overline{V_d}, \overline{V_i}$ These are called the symmetrical components of the voltage vectors $\overline{V_1}, \overline{V_2}, \overline{V_3}$.

✓ **Symmetrical Components of Current**

The same approach can be applied to decompose the current vectors, in order to obtain:

$$\begin{cases} \overline{I_1} = \overline{I_h} + \overline{I_d} + \overline{I_i} \\ \overline{I_2} = \overline{I_h} + a^2\overline{I_d} + a\overline{I_i} \\ \overline{I_3} = \overline{I_h} + a\,\overline{I_d} + a^2\overline{I_i} \end{cases} \qquad \mathbf{V.21}$$

In Matrix Form:

$$\begin{bmatrix} \overline{I_1} \\ \overline{I_2} \\ \overline{I_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{I_h} \\ \overline{I_d} \\ \overline{I_i} \end{bmatrix} \qquad \mathbf{V.22}$$

➢ **Inverse Transformation :**

$$\begin{bmatrix} \overline{I_h} \\ \overline{I_d} \\ \overline{I_i} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \overline{I_1} \\ \overline{I_2} \\ \overline{I_3} \end{bmatrix} \Rightarrow \begin{matrix} \overline{I_h} = \frac{1}{3}(\overline{I_1} + \overline{I_2} + \overline{I_3}) \\ \overline{I_d} = \frac{1}{3}(\overline{I_1} + a\overline{I_2} + a^2\overline{I_3}) \\ \overline{I_i} = \frac{1}{3}(\overline{I_1} + a^2\overline{I_2} + a\overline{I_3}) \end{matrix} \qquad \mathbf{V.23}$$

V.9. Construction of Sequence Networks

The sequence network illustrates the flow of sequence currents when they occur in an electrical system. To represent this phenomenon correctly, it is necessary to interconnect the networks of the different sequence components. The construction of these networks is done from the fault point, that is, the location where the imbalance originates in the system, whether it comes from an electrical fault or a load imbalance.

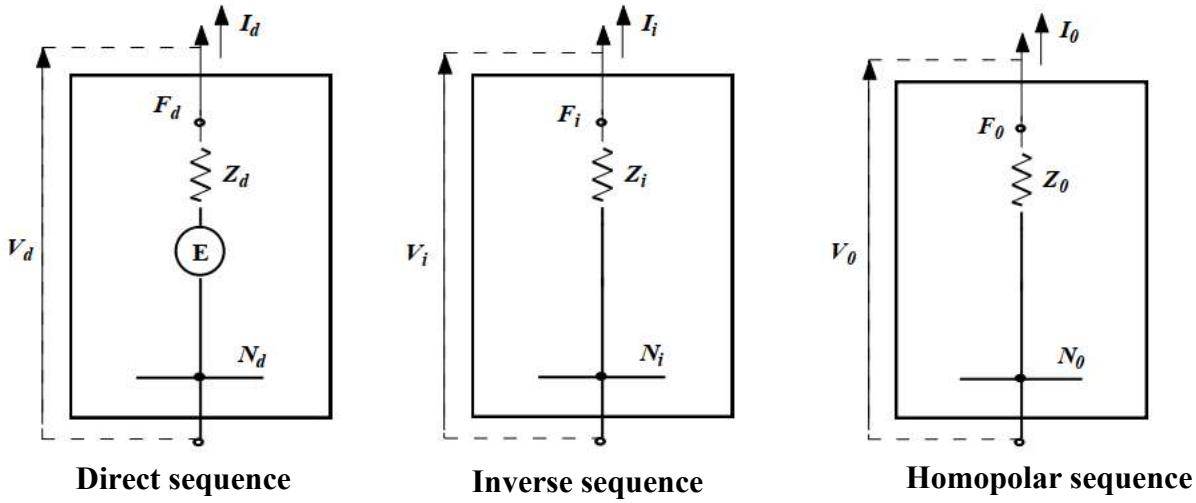


Figure V.12. Representation of the Network in Positive, Negative, and Zero Sequences.

- ☞ In the sequence networks, voltages are taken as line-to-neutral voltages.
- ☞ The positive-sequence network is the only active one, as it contains the voltage source.
- ☞ Phase a voltage is chosen as the reference, and the voltages of the other two phases are expressed relative to this reference, as shown in Figure V.12.

As previously indicated, only the positive-sequence network has a voltage source, which is the Thevenin equivalent. With this convention, appropriate signals must be assigned to the sequence voltages:

$$\begin{cases} E = V_d + Z_d I_d \\ 0 = V_i - Z_i I_i \\ 0 = V_h - Z_h I_h \end{cases} \Rightarrow \begin{cases} V_d = E - Z_d I_d \\ V_i = -Z_i I_i \\ V_h = -Z_h I_h \end{cases} \qquad \textbf{V.24}$$

V.9.1. Single-Phase Short Circuit

Phase a is faulty in Figure V.13. Since the load current is neglected, the currents in phases b and c are zero, and the voltage at the fault point is $V_a = I_a Z_f$. The sequence components of the currents are given by:

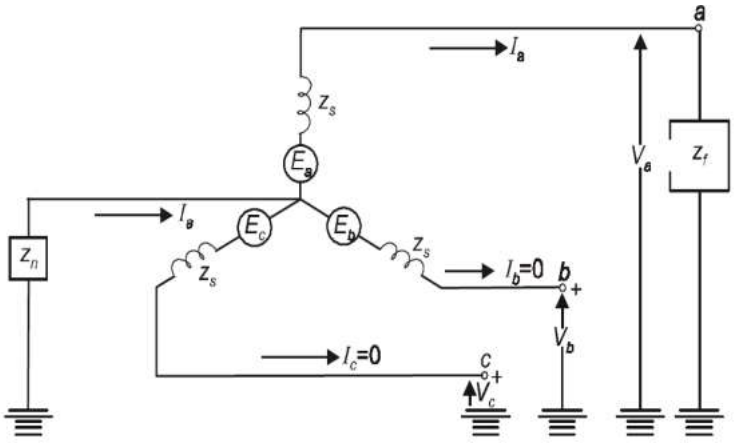


Figure V.13. Phase-to-Ground Short Circuit at Phase a.

$$\begin{cases} I_b = 0 \\ I_C = 0 \\ V_a = Z_f \cdot I_a \end{cases} \quad \textbf{V.25}$$

$$\begin{cases} \bar{I}_h = \frac{1}{3}(\bar{I}_a + \bar{I}_b + \bar{I}_c) \\ \bar{I}_d = \frac{1}{3}(\bar{I}_a + a\bar{I}_b + a^2\bar{I}_c) \\ \bar{I}_i = \frac{1}{3}(\bar{I}_a + a^2\bar{I}_b + a\bar{I}_c) \end{cases} \quad \textbf{V.26}$$

By substituting equation (V.25) into (V.26), we obtain:

$$\begin{cases} \bar{I}_h = \frac{1}{3}(\bar{I}_a) \\ \bar{I}_d = \frac{1}{3}(\bar{I}_a) \Rightarrow \bar{I}_h = \bar{I}_d = \bar{I}_i = \frac{1}{3}(\bar{I}_a) \\ \bar{I}_i = \frac{1}{3}(\bar{I}_a) \end{cases} \quad \textbf{V.27}$$

$$\bar{V}_a = \bar{V}_h + \bar{V}_h + \bar{V}_i = Z_f \cdot I_a = 3Z_f I_d \quad \textbf{V.28}$$

Which gives:

$$I_d = 3\frac{E}{Z_d + Z_i + Z_h + 3Z_f} \quad \textbf{V.29}$$

From these equations, we can deduce that all three sequence circuits carry the same current, so they are in series through the fault impedance 3Zf, as illustrated in Figure V.14.

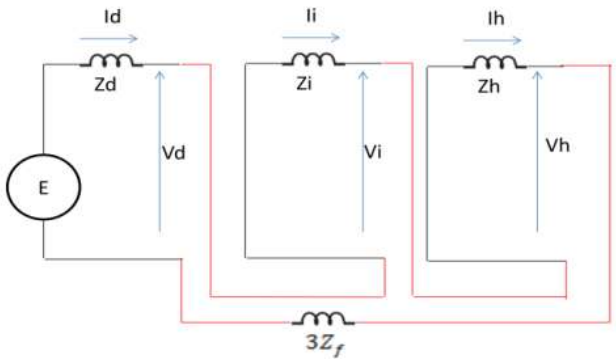


Figure V.14. Sequence Circuit in the Presence of a Phase-to-Ground Short Circuit.

The short-circuit current Ia is:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 3I_h \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3\dfrac{E}{Z_d + Z_i + Z_h + 3Z_f} \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{V.30}$$

- **Phase Voltages**

$$\begin{cases} V_d = E\dfrac{Z_i + Z_h + 3Z_f}{Z_d + Z_i + Z_h + 3Z_f} \\ V_i = E\dfrac{-Z_i}{Z_d + Z_i + Z_h + 3Z_f} \\ V_h = E\dfrac{-Z_h}{Z_d + Z_i + Z_h + 3Z_f} \end{cases} \qquad \mathbf{V.31}$$

V.9.2. Two-Phase Short Circuit

Figure V.15 shows a line-to-line fault. A short circuit occurs between phases b and c through a fault impedance Zf. The fault current flows between phases b and c, returning to the source through phase b and returning through phase c.

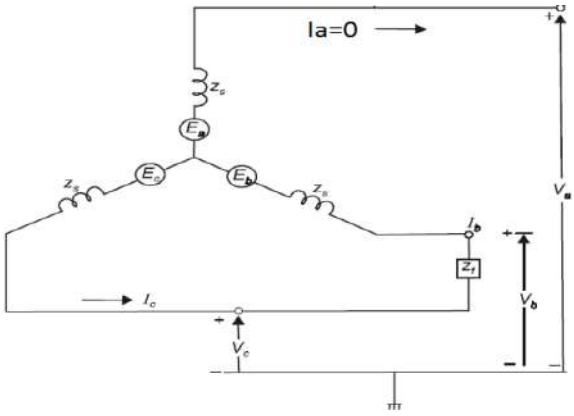


Figure V.15. Phase-to-Phase Short Circuit Between Phases b and c (Two-Phase, Without Ground)

$$\begin{cases} I_a = 0 \\ I_b + I_C = 0 \\ V_b - V_C = Z_f \cdot I_b \end{cases} \quad \mathbf{V.32}$$

$$\begin{cases} \bar{I}_h = \frac{1}{3}(\bar{I}_a + \bar{I}_b + \bar{I}_c) \\ \bar{I}_d = \frac{1}{3}(\bar{I}_a + a\bar{I}_b + a^2\bar{I}_c) \\ \bar{I}_i = \frac{1}{3}(\bar{I}_a + a^2\bar{I}_b + a\bar{I}_c) \end{cases} \quad \mathbf{V.33}$$

By substituting equation (V.18) into (V.19), we obtain

$$\begin{cases} \bar{I}_h = 0 \\ \bar{I}_d = \frac{1}{3}(a - a^2)\bar{I}_b \\ \bar{I}_i = \frac{1}{3}(a^2 - a)\bar{I}_b \end{cases} \Rightarrow \begin{cases} \bar{I}_h = 0 \\ \bar{I}_d = -\bar{I}_i \end{cases} \quad \mathbf{V.34}$$

From V.20, we can say that the zero-sequence circuit is open, while the positive-sequence and negative-sequence circuits are in series.

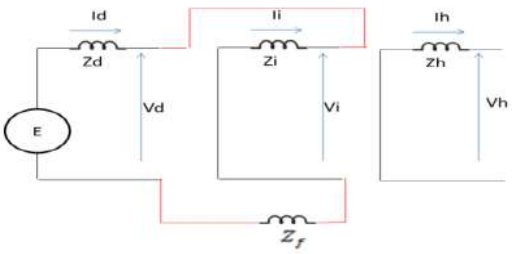


Figure V.16. Sequence Circuit in the Presence of a Phase-to-Phase Short Circuit (Without Ground)

➢ **Phase Currents**

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{I_h} \\ \overline{I_d} \\ \overline{I_i} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ \dfrac{E}{Z_d + Z_i + Z_f} \\ \dfrac{-E}{Z_d + Z_i + Z_f} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{E(a^2 - a)}{Z_d + Z_i + Z_f} \\ \dfrac{E(a - a^2)}{Z_d + Z_i + Z_f} \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{-j\sqrt{3}E}{Z_d + Z_i + Z_f} \\ \dfrac{j\sqrt{3}E}{Z_d + Z_i + Z_f} \end{bmatrix} \qquad \mathbf{V.35}$$

➢ **Phase Voltages**

$$\begin{cases} V_d = \dfrac{E(Z_i + Z_f)}{Z_d + Z_i + Z_f} \\ V_i = \dfrac{E(Z_i + Z_f)}{Z_d + Z_i + Z_f} \\ V_h = 0 \end{cases} \qquad \mathbf{V.36}$$

Two-Phase-to-Ground Short Circuit

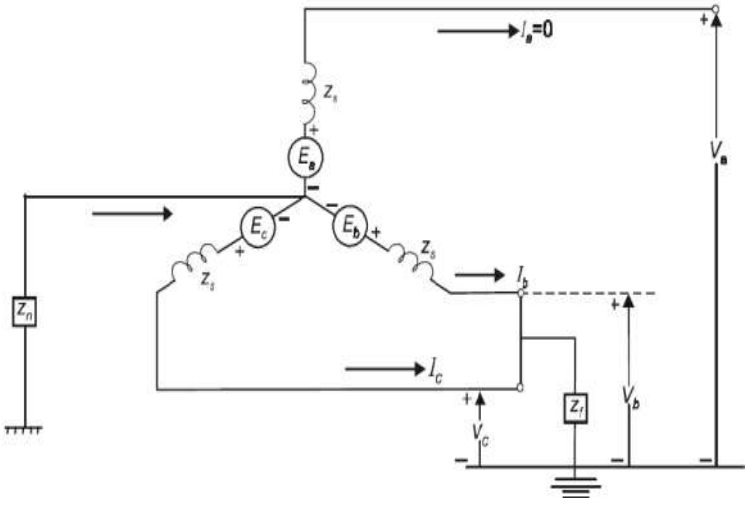


Figure V.17. Phase-to-Phase-to-Ground Short Circuit Between Phases b and c (Two-Phase-to-Ground).

This is an asymmetrical fault. Suppose that phases b and c are short-circuited with each other and with the ground through an impedance Zf, for which the boundary conditions can be written as follows:

$$\begin{cases} I_a = 0 \\ V_b = V_C = Z_f(I_b + I_C) \end{cases} \quad \textbf{V.37}$$

$$\begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{bmatrix} \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_b} \end{bmatrix} \quad \textbf{V.38}$$

$$\begin{cases} V_h = \frac{1}{3}(V_a + 2V_b) \\ V_d = V_i = \frac{1}{3}(V_a + aV_b + a^2V_C) = \frac{1}{3}(V_a - V_b) \end{cases} \quad \textbf{V.39}$$

$$V_h - V_d = \frac{1}{3}(V_a + 2V_b - V_a + V_b) \rightarrow V_b = Z_f(I_b + I_C) \quad \textbf{V.40}$$

$$V_h - V_d = Z_f(I_b + I_C) \quad \textbf{V.41}$$

$$\begin{cases} \overline{I_h} = \frac{1}{3}(\overline{I_a} + \overline{I_b} + \overline{I_c}) \\ \overline{I_d} = \frac{1}{3}(\overline{I_a} + a\overline{I_b} + a^2\overline{I_c}) \Rightarrow \overline{I_h} = \frac{1}{3}(\overline{I_b} + \overline{I_c}) \Rightarrow \overline{I_h} = \frac{V_h - V_d}{3Z_f} \\ \overline{I_i} = \frac{1}{3}(\overline{I_a} + a^2\overline{I_b} + a\overline{I_c}) \end{cases} \quad \textbf{V.42}$$

$$V_h - V_d = 3Z_f I_h \quad \textbf{V.43}$$

According to equations (V.41) and (V.43), we can say that the three sequences are in parallel, as shown in the following diagram:

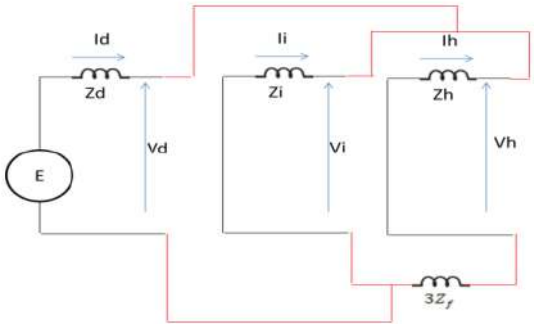


Figure V.18. Sequence Circuit in the Presence of a Two-Phase-to-Ground Short Circuit.

V.9.3. Three-Phase Short Circuit

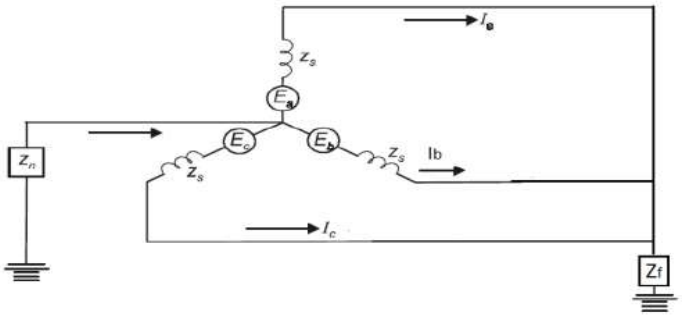


Figure V.19. Three-Phase Short Circuit.

Here, we consider a symmetrical fault. Phases a, b, and c are short-circuited with each other as well as with the ground through an impedance Zf. The boundary conditions can then be formulated as follows:

$$V_a = V_b = V_c \qquad \mathbf{V.44}$$

$$\begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{bmatrix} \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{V_a} \\ \overline{V_a} \\ \overline{V_a} \end{bmatrix} \qquad \mathbf{V.45}$$

$$\begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} = \begin{bmatrix} \overline{V_a} \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{V.46}$$

$$\begin{cases} E = V_d + Z_d I_d \\ 0 = V_i + Z_i I_i \\ 0 = V_h + Z_h I_h \end{cases} \Rightarrow \begin{cases} E = 0 + Z_d I_d \\ 0 = 0 + Z_i I_i \\ 0 = V_a + Z_h I_h \end{cases} \Rightarrow \begin{cases} I_d = \frac{E}{Z_d} \\ I_i = 0 \\ I_h = 0 \end{cases} \qquad \mathbf{V.47}$$

According to equation (V.35), we can conclude that the three sequences are independent of each other, and furthermore, the negative-sequence and zero-sequence currents are zero, as shown in the following diagram:

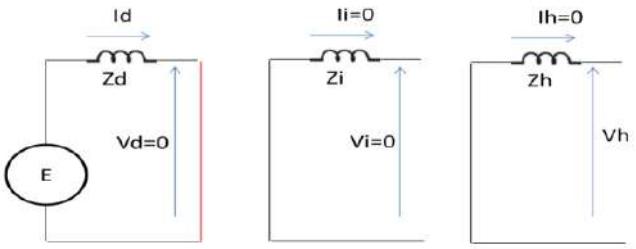


Figure V.20. Sequence Circuit in the Presence of a Three-Phase Short Circuit.

- **Phase Currents**

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{I_h} \\ \overline{I_d} \\ \overline{I_i} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ \dfrac{E}{Z_d} \\ 0 \end{bmatrix} \qquad \mathbf{V.48}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} \dfrac{E}{Z_d} \\ a^2 \dfrac{E}{Z_d} \\ a \dfrac{E}{Z_d} \end{bmatrix} \qquad \mathbf{V.49}$$

- **Phase Voltages**

$$\begin{bmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{V_h} \\ \overline{V_d} \\ \overline{V_i} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{V.50}$$

$$\begin{bmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_C} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{V.51}$$

V.10. Equivalent Impedances of Network Elements

The calculation of fault currents requires first determining the equivalent sequence impedances (positive, negative, and zero) as seen from the fault point for all the equipment making up the electrical circuit. These values must be provided by the manufacturer.

- ☞ Zd represents the positive-sequence impedance of an element. It corresponds to the impedance measured when a balanced three-phase voltage system is applied across the

three phases. This impedance is identical to the short-circuit impedance Zcc used for calculating currents during a symmetrical three-phase short circuit, hence: $Z_d = Z_{CC}$

- ☞ Due to the symmetrical nature of cables, lines, and transformers, the positive-sequence impedance is identical to the negative-sequence impedance for these elements. Thus, we have:: $Z_i = Z_d = Z_{CC}$.
- ☞ For electrical machines, whether static or rotating, impedance values are rarely given in ohms. They are generally referenced to the machine's rated power and voltage. These impedances are then expressed in per-unit values, either as a percentage (%) or in per unit (pu), obtained from the ohmic values using the following relationships:

$$Z(pu) = Z\frac{S_n}{U_n^2}\ , Z(\%) = 100\ Z\frac{S_n}{U_n^2} \qquad \mathbf{V.52}$$

Where,

U_n : Rated Voltage of the Machine (kV),

S_n :Rated Power of the Machine (MVA)

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Exercises and Solved Problems

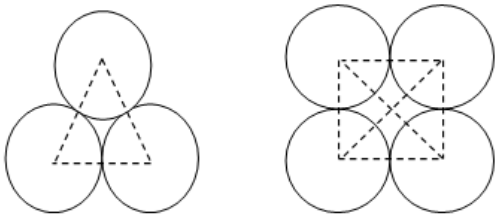
Calculation of Power Line Parameters

Exercise 01 :

Find the geometric mean radius GMR of a conductor as a function of the radius r of a single strand (brin) for :

(a) Three identical strands in figure (a)

(b) Four identical strands in figure (b)



(a) (b)

Solution:

(a)

$GMR = \sqrt[9]{(r'2r \times 2r)^3}$

$\Longrightarrow GMR = \sqrt[3]{re^{-1/4} \times 2r \times 2r}$

$\Longrightarrow GMR = 1.46\, r$

(b)

$GMR = \sqrt[16]{(r'2r \times 2r \times \sqrt{2} \times 2r)^4}$

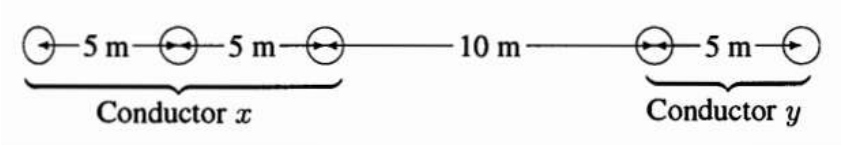
$\Longrightarrow GMR = \sqrt[4]{e^{-1/4} \ . \ 8\sqrt{2}r^4}$

$\Longrightarrow GMR = 1.723\, r$

Exercise 02 :

A single-phase transmission line circuit is composed of three solid wires with a radius of 0.5 cm. The return circuit is composed of two solid 2.5 cm radius wires.

The arrangement of the conductors is shown in the figure. Applying the concept of GMD and GMR, find the total inductance of the line in mH /km.



Solution :

$$D_m = \sqrt[6]{20 \times 25 \times 15 \times 20 \times 10 \times 15}$$

$$\Longrightarrow D_m = 16.802\, m$$

$$R_{SX} = \sqrt[9]{(e^{-1/4} \times 0.005 \times 5 \times 10)(e^{-1/4} \times 0.005 \times 5 \times 5)(e^{-1/4} \times 0.005 \times 5 \times 10)}$$

$$\Longrightarrow R_{SX} = 0.5366\, m$$

$$R_{SY} = \sqrt[4]{(e^{-1/4} \times 0.025 \times 5)^2}$$

$$\Longrightarrow D_{SY} = 0.312\, m$$

So :

$$L = 0.2 \ln \frac{GMD}{GMR} \text{ mH/km}$$

$$L_X = 0.2ln\frac{16.802}{0.5366} \Longrightarrow L_X = 0.6888 \text{ mH/km}$$

$$L_Y = 0.2ln\frac{16.802}{0.312} \Longrightarrow L_X = 0.79725 \text{ mH/km}$$

The total inductance is:

$$L = L_X + L_Y = 0.6888 + 0.79725$$

$$\Longrightarrow L = 1.48605\ mH/km$$

Exercise 03:

A 69 Kv, three-phase short transmission line is 16 km long operating. The line has a per phase series impedance of 0.125 + j0.4375 Ω/ km.

1. Trace the model of the line.

Determine the the sending end voltage VS, voltage regulation, the sending end power and the the transmission efficiency, when the line delivers:

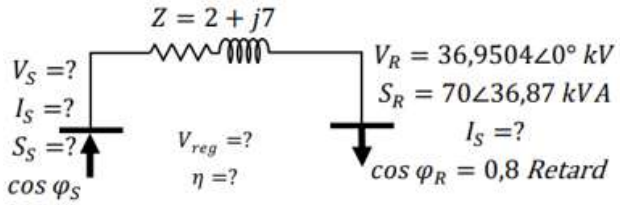
(a) 70 MVA, 0.8 lagging power factor at 64 kV.

(b) 120 MW , unity power factor at 64 kV.

Solution 03 :

Total line impedance is :

$$z = (0.125 + j0.4375)(16) = 2 + j7\Omega$$



The receiving end voltage per phase is :

$$V_R = \frac{64\angle 0^\circ}{\sqrt{3}} = 36.9504\angle 0^\circ \text{ kV}$$

(a) **The receiving end apparent power is :**

$$S_{R(3\Phi)} = 70\angle cos^{-1}\, 0.8 = 70\angle 36.87^\circ = 56 + j42\ MVA$$

The receiving end currenr per phase is given by

$$I_R = \frac{S^*_{R(3\Phi)}}{3V^*_R} = \frac{70000\angle -36.87^\circ}{3 * 36.9504\angle 0^\circ} = 631.477\angle -36.87\ A$$

The sending end voltage :

$$V_S = V_R + Z\ \ I_R = 36.9504\angle\, 0^\circ\ +\ (2\ +\ j7)(631.477\angle\, 36.87^\circ)(10^{-3})$$

$$=\ 40.708\angle 3.9137^\circ KV$$

The sending end line to line voltage

$$\left|V_{S(L-L)}\right| =\ \sqrt{3}\ |V_S| =\ 70.508\ \text{kV}$$

The sending end power

$$S_{S(3\Phi)} = 3\ V_S.\ I^*_S = 3\ \times\ 40.7086\ \angle\ 3.9137\ \times\ 631.4776\ \angle\ 36.87^\circ\ \times\ 10^{-3}$$

$$=\ 58.393\ MW\ +\ j50.374\ Mvar$$

$$= 77.1185\angle 40.7837^\circ MVA$$

Voltage regulation is

$$VR\ (\%) = \frac{70.508 -\ 64}{64} \times 100 = 10.169\%$$

Transmission line efficiency is :

$$\eta = \frac{P_{R(3\Phi)}}{P_{S(3\Phi)}} = \frac{56}{58.393} \times 100 = 95.90\%$$

(b) **The receiving end apparent power is**

$$S_{R(3\Phi)} == 120\angle 0° = 120 + j0\ MVA$$

The receiving end current

$$I_R = \frac{S^*_{R(3\Phi)}}{3V^*_R} = \frac{12000\angle 0°}{3 * 36.9504\angle 0°} = 1082.53\angle 0° A$$

The sending end voltage

$$V_S = V_R + Z\ \ I_R = 36.9504\angle\ 0°\ +\ (2\ +\ j7)(1082.53\angle 0°)(10^{-3})$$

$$=\ 39.8427\angle 10.9639\ kV$$

The sending end line to line voltage

$$\left|V_{S(L-L)}\right| =\ \sqrt{3}\ |V_S| =\ 69.0096\ \text{kV}$$

The sending end power

$$S_{S(3\Phi)} = 3\ V_S.\ I_S^* = 3\ \times\ 39.8427\angle 10.9639\ \times\ 1082.53\angle 0°\ \times\ 10^{-3}$$

$$=\ 127.031\ MW\ +\ j24.609\ Mvar$$

$$= 129.393\angle 10.9639° MVA$$

Voltage regulation is

$$VR\ (\%) = \frac{69.0096 - 64}{64} \times 100 = 7.8275\%$$

Transmission line efficiency is :

$$\eta = \frac{P_{R(3\Phi)}}{P_{S(3\Phi)}} = \frac{120}{127.031} \times 100 = 94.465\%$$

Exercise 04 :

A 230 kV three-phase transmission line, 80 km long, has the characteristics below:

Longitudinal impedance Z=0.05 + j0.45 Ω/km

The transverse admittance Y = $\mathrm{j}3.4 * 10^{-6}$ S/km

Using the π model, determine:

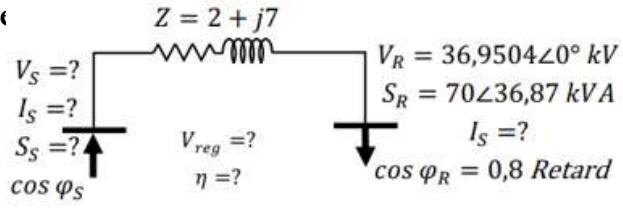
1. The transmission line ABCD constant.
2. Find the sending end voltage and current, voltage regulation, the sending end power and the transmission efficiency when the line delivers:

(a) A 200 MVA, 0.8 lagging power factor at 220 kV.

(b) A 306 MW, unity power factor at 220 kV.

Solution 04:

Total line impedance and shunt admission are



Z =(0.05 + j 0.45) (80) = 4 + j 36 Ω

Y= (j 3.4*10^{-6}) (80)= j 0.272*10^{-3} Siemens

The ABCD constants of the π model are

$$A = D = \left(1 + \frac{ZY}{2}\right) = (1 + \frac{(4 + j36)(j0.272 * 10^{-3})}{2})$$

$$= 0.9951 + j0.000544$$

$$B = Z = 4 + j36$$

$$C = Y\left(1 + \frac{ZY}{4}\right)$$

$$C = (j0.272 * 10^{-3})(1 + \frac{(4 + j36)(j0.272 * 10^{-3})}{4})$$

$$= j0.0002713$$

The receiving end voltage per phase is :

$\mathrm{V_R} = \frac{220\angle 0°}{\sqrt{3}}$ =127∠0° kv

(a) **The receiving end apparent power is :**

$$S_R(3\Phi) = 200\angle cos^{-1}(0.8) = 200\angle 36.87° = 160 + j120\ MVA$$

The current per phase at the line output is given by:

$$I_R = \frac{S_R{}^*(3\Phi)}{3V_R{}^*} = \frac{200000\angle -36.87°}{3*127\angle 0°} = 524.864\angle -36.87°A$$

The sending end voltage

$$V_s = AV_R + BI_R \quad (1)$$

$$V_s = (0.9951 + j0.000544)(127\angle 0°) + (4 + j36)(524.864 * 10^{-3}\angle -36.87°)$$

$$= 140.1051\angle 5.704°\ kV$$

The sending end line to line voltage

$$\left|V_{S(L-L)}\right| = \sqrt{3}\ |V_s| = 242.67\ \text{kV}$$

The current per phase at the line input is given by:

$$I_S = C\ V_R + D\ I_R$$

$$I_S = (j0.0002713)(127000\angle 0°) + (0.9951 + j0.000544)(524.864\angle -36.87°)$$

$$= 502.38\angle -33.69°\ A$$

The sending end power

$$S_s(3\Phi) = 3V_S\ I_S{}^*$$

$$= 3 * 140.1051\angle 5.704° * 502.38\angle 33.69° * 10^{-3}$$

$$= 163.1795MW + j39.396\ MVA$$

$$= 211.16\angle 39.396°MVA$$

Voltage regulation (VR) is

$$RT(\%) = \frac{\left|V_{R(avide)}\right| - \left|V_{R(encharge)}\right|}{\left|V_{R(encharge)}\right|} \times 100$$

A vide $I_R = 0$ and equation (1) $\Rightarrow$ $V_{R(avide)} = \frac{V_S}{A}$

$$RT = \frac{\frac{V_S}{A} - V_R}{V_R} * 100\% = \frac{\frac{242.67}{0.9951}kV - 220kV}{220kV} * 100\%$$

$$= 10.847\%$$

Transmission line efficiency is :

$$\eta = \frac{P_R(3\Phi)}{P_S(3\Phi)} = \frac{160}{163.179} * 100 = 98.052\%$$

The receiving end apparent power is :

(a) $S_R(3\Phi) = 300\angle cos^{-1}(0) = 200\angle 36.87° = 306 + j0\ MVA$

The current per phase at the line output is given by:

$$I_R = \frac{S_R{}^*(3\Phi)}{3V_R{}^*} = \frac{306000\angle 0°}{3*127\angle 0°} = 803.402\angle 0° A$$

The voltage at the line input is:

$$V_s = AV_R + BI_R$$

$$(1) V_s = (0.9951 + j0.000544)(127\angle 0°) + (4 + j36)(803.402 * 10^{-3}\angle 0°)$$

$$= 132.807\angle 12.6°\ kV$$

The voltage module (between lines) at the line input

$$\left|V_{S(L-L)}\right| = \sqrt{3}\ |V_s|$$

$$= \sqrt{3}\ 132.807 = 230.029 \text{ kV}$$

The current per phase at the line input is given by:

$$I_S = C\ V_R + D\ I_R$$

$$I_S = (j0.0002713)(127000\angle 0°) + (0.9951 + j0.000544)(803.402 * 10^{-3}\angle 0°)$$

$$= 799.862\angle 2.5° A$$

The sending end power

$$S_s(3\Phi) = 3V_S\ I_S{}^*$$

$$= 3 * 132.807\angle 12.6° * 799.862\angle -2.5° * 10^{-3}$$

$$= 313.742\ MW + j55.9\ MVA$$

$$= 318.68\angle 10.1°\ MVA$$

Voltage regulation (VR) is

$$RT(\%) = \frac{\left|V_{R(avide)}\right| - \left|V_{R(encharge)}\right|}{\left|V_{R(encharge)}\right|} \times 100$$

A vide $I_R = 0$ and equation (1) $\Rightarrow$ $V_{R(avide)} = \frac{V_S}{A}$

$$VR = \frac{\frac{V_S}{A} - V_R}{V_R} * 100\% = \frac{\frac{230.029}{0.9951} kV - 220kV}{220kV} * 100\%$$

$$= 5.073\%$$

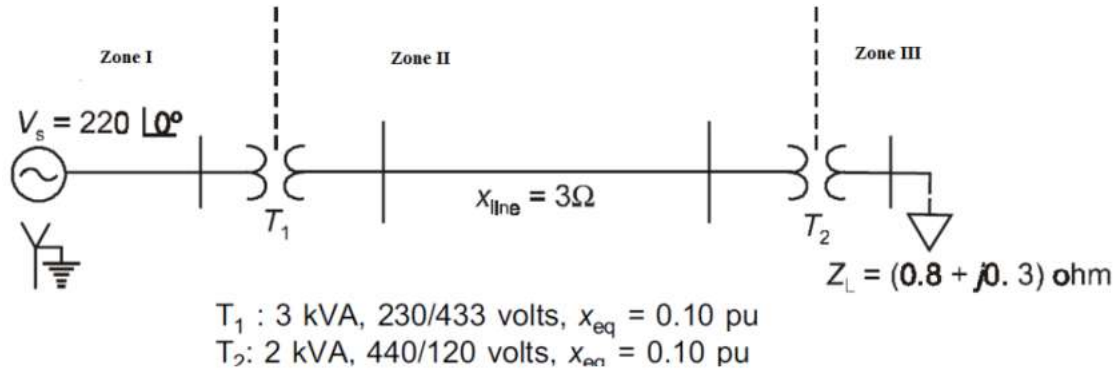
Transmission line efficiency is :

$$\eta = \frac{P_R(3\Phi)}{P_S(3\Phi)} = \frac{306}{313.742} * 100 = 97.53\%$$

Per Unit system and short-circuit current calculations

Exercise 05:

Consider the following single-line diagram of a single-phase network:



Using 3 KVA and 230V as a basis;

- Draw the impedance diagram,

- Calculate the current in the load in PU and amperes.

Solution 05:

Base table

	S (KVA)	V (V)	Z (Ω)	I (A)
Zone 1	3	$V_{b1} = 230$	$Z_{b1} = \frac{230^2}{3000} = 17.63$	$I_{b1} = \frac{3000}{230} = 13.04$
Zone 2	3	$V_{b2} = 230.\frac{433}{230} = 433$	$Z_{b2} = \frac{433^2}{3000} = 62.49$	$I_{b2} = \frac{3000}{433} = 5.92$
Zone 3	3	$V_{b3} = 433.\frac{120}{440} = 118.09$	$Z_{b3} = \frac{118.09^2}{3000} = 5.64$	$I_{b3} = \frac{3000}{118.09} = 25.4$

1. Impedance diagram:

- Generator: negligible impedance.

- Transformer T1: No change at the base level → $ZT1Pu,N = ZT2Pu,A$

→ $ZT1 = j0.1\ pu$

- Line: : $ZL_{PU} = \frac{Z_L}{Z_{b2}} = \frac{3}{62,49} \rightarrow ZL = 0.048\ pu$

- Transformer T2:

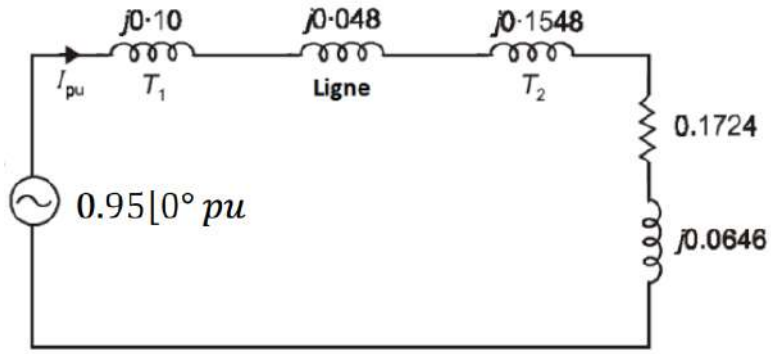
$$Z_{T2Pu,N} = Z_{T2Pu,A}\frac{U_{b,A}{}^2}{U_{b,N}{}^2}\frac{S_{b,N}}{S_{b,A}} = j0.1\frac{440^2}{433^2}\frac{3000}{2000}$$ → $ZT2 = j0.154\ pu$

-Load :

$$Z_{\text{Load } PU} = \frac{Z_{\text{Load}}}{Z_{b3}} = \frac{0.8 + j0.3}{5.64}$$

$Z_{\text{Load}} = 0.172 + j0.064\ pu$

$$V_{Spu} = \frac{V_S}{V_{b1}} = \frac{220\lfloor 0°}{230} = 0.95\lfloor 0°\ pu$$



2.Calculation of the current in the load:

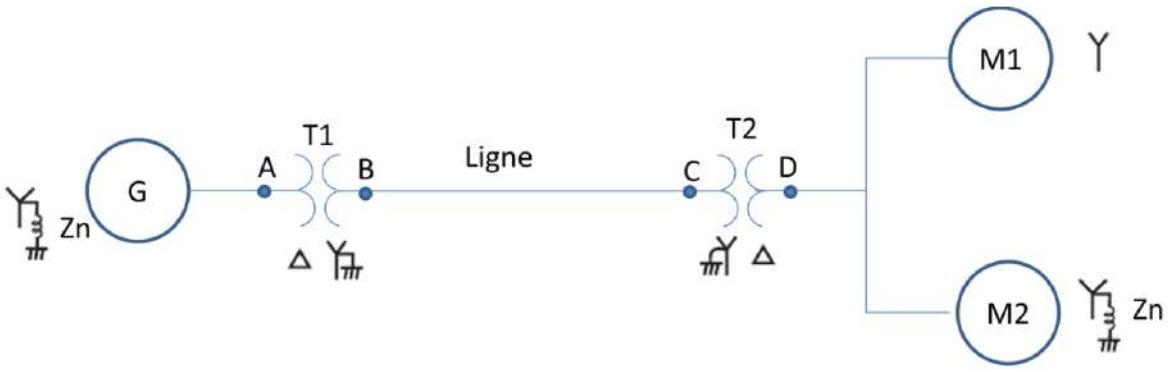
$$I_{PU} = \frac{0.95\lfloor 0°}{j(0.1 + 0.048 + 0.154 + 0.064) + 0.172} = \frac{0.95\lfloor 0°}{0.4058\lfloor 65.86°}$$

$I_{PU} = 2.355\lfloor -65.86°\ pu$

I Load $= IPU \times Ib3 = 2.355\lfloor -65.86° \times 25.4 \rightarrow Ich = 59.83\lfloor -65.86°\ A$

Exercise 06 (single-phase asymmetrical fault)

Consider the following single-line diagram:



1. Using 50 MVA and 11 kV as a basis, draw the impedance diagram for the three sequences:

G1 : 50 MVA, 11 kV, X_{G1}=20%. X_0=6%. Xn=2.5 Ω.

M1 : 30 MVA, 10 kV, X_{M1}=25%. X_0=6%

M2: 15 MVA, 10 kV, X_{M2}=25%. X_0=6% ; Xn=2.5 Ω.

T1 : 60 MVA, 10.8 kV – 121 kV, X_{T1}=10%.

T2 : 60 MVA, 121 kV -10.8 kV, X_{T2}=10%.

Line : X_l=100 Ω, X_0=300 Ω.

2. A single-phase fault has occurred at node D. Calculate the short-circuit current.

Solution 06:

Base table

	S (MVA)	**V (KV)**	**Z (Ω)**
Zone 1	50	$V_{b1} = 11$	$Z_{b1} = \frac{11^2}{50} = 2.42$
Zone 2	50	$V_{b2} = 11 \times \frac{121}{10.8} = 123.2$	$Z_{b2} = \frac{123.2^2}{50} = 303.56$
Zone 3	50	$V_{b3} = 123.2.\frac{10.8}{121} = 11$	$Z_{b3} = \frac{11^2}{50} = 2.42$

1. Impedance diagram:

- Generator G1:

$Z_d\ G1 = ZiG1 = j0.2\ pu$

$Z_h\ G1 = j0.06\ pu$

$Z_{nG1} = \frac{2.5}{2.42}$

$\rightarrow ZnG1 = j1.033pu$

- T2 transformer:

$Z_{dT2} = Z_{iT2} = Z_{hT2} = j0.1\ \frac{50}{60}\frac{10.8^2}{11^2}$

$= j0.0803\ pu$

- Line :

$$Z_{dL} = Z_{iL} = \frac{Z_L}{Z_{b2}} = \frac{j100}{303.56}$$

$= j0.329\ pu$

$$Z_{hL} = \frac{Z_{Lh}}{Z_{b2}} = \frac{j300}{303.56}$$

$= j0.988\ pu$

- M1:

$$Z_{d\,M1} = Z_{iM1} = j0.25\frac{50}{30}\frac{10^2}{11^2}$$

$= j0.344\ pu$

$$Z_{h\,M1} = j0.06\ \frac{50}{30}\frac{10^2}{11^2}$$

$= j0.082\ pu$

- M2 :

$$Z_{d\,M2} = Z_{iM2} = j0.25\frac{50}{15}\frac{10^2}{11^2}$$

$= j0.688\ pu$

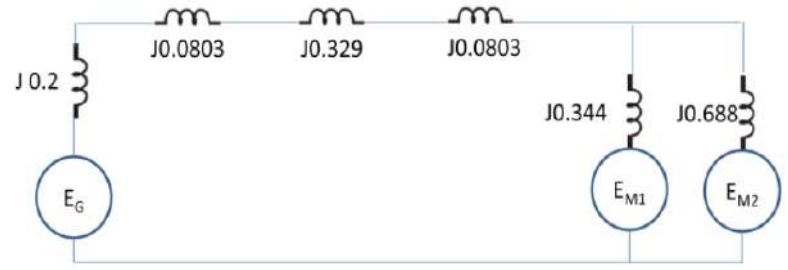
$$Z_{h\,M2} = j0.06\ \frac{50}{15}\frac{10^2}{11^2}$$

$= j0.164\ pu$

$$Z_{nM2} = \frac{2.5}{2.42}$$

$\rightarrow ZnM2 = j1.033pu$

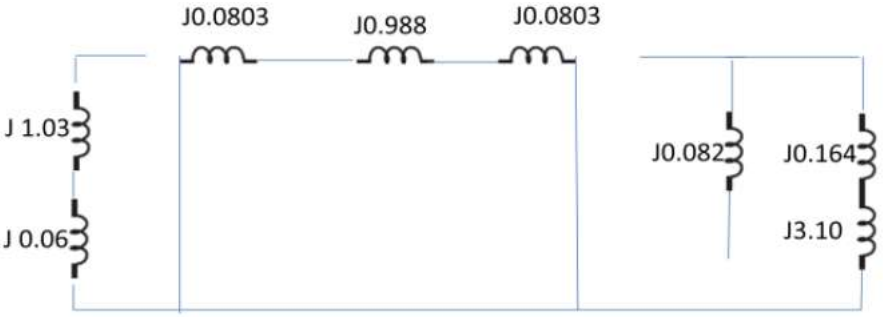
• **Positive sequence:**



• **Negative sequence:**

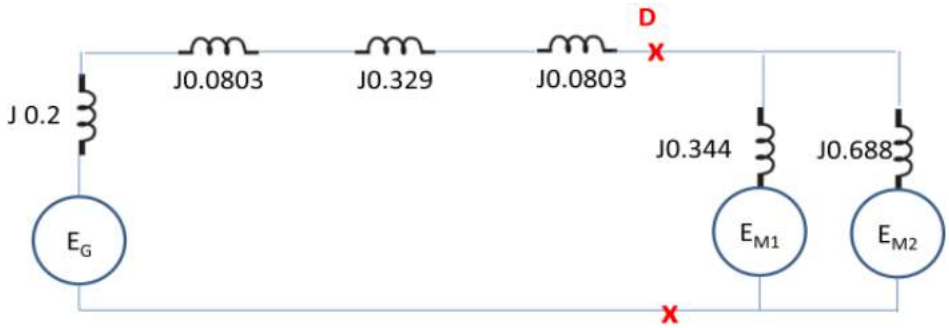


• **Zero sequence:**



2.Homopolar fault in the motors (at point D):

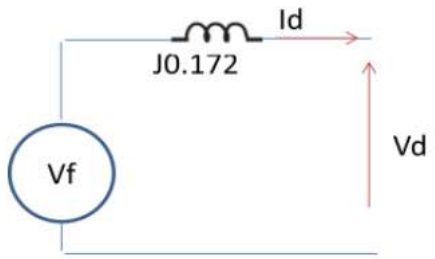
- Direct sequence diagram:



$Z_{th,\,D}$ = (j0.2 + j0.0803 + j0.329 + j0.0803)//(j0.344 //j0.688)

$Z_{thD} = j0.172\ pu$

$V_f = 10/\ 11 = 0.909\ pu$

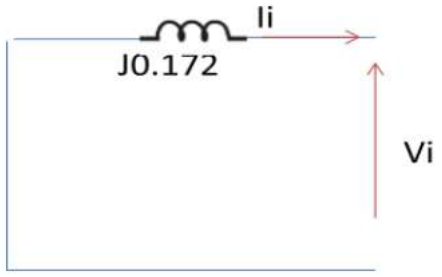


- **Reverse sequence diagram :**



$ZthI = (j0.2 + j0.0803 + j0.329 + j0.0803)//(j0.344\ //j0.688)$

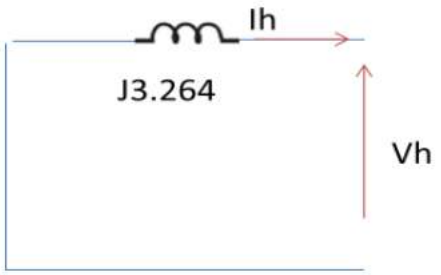
$ZthI = j0.172\ pu$



- Diagram of the homopolar sequence:



$ZthH = j0.164 + j3.10 = j3.264\ pu$



For a single-phase fault, the three sequences are in series, the forward, reverse, and zero-sequence currents are equal and equal to:

$$I_d = I_i = I_h = \frac{V_f}{Z_d + Z_i + Z_h} = \frac{0.909}{j3.264 + j0.172 + j0.172}$$

$= -0.252\ pu$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 3\frac{E}{Z_d+Z_i+Z_h} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.341 \\ 0 \\ 0 \end{bmatrix} pu$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -1.341 \\ 0 \\ 0 \end{bmatrix} \times I_{b3} = \begin{bmatrix} -3.52 \\ 0 \\ 0 \end{bmatrix} \quad KA$$